

School - Level
Trigonometry

Topic 18
Trigonometric
Circular Functions

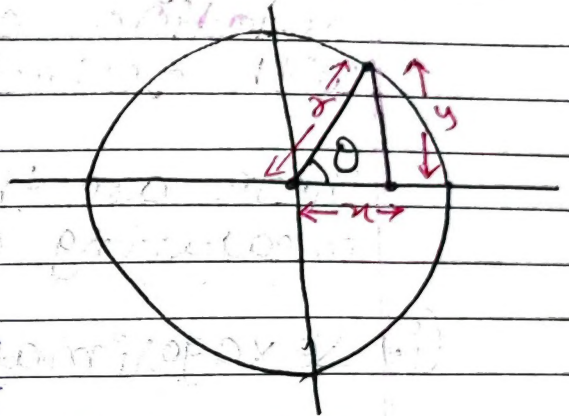
Concept 1: Trigonometric Ratios for any angle.

इस trigonometric Ratios होते हैं।

★ $\sin(\theta) = \frac{y}{r}$ $\cot(\theta) = \frac{x}{y}$

$\cos(\theta) = \frac{x}{r}$ $\sec(\theta) = \frac{r}{x}$

$\tan(\theta) = \frac{y}{x}$ $\csc(\theta) = \frac{r}{y}$



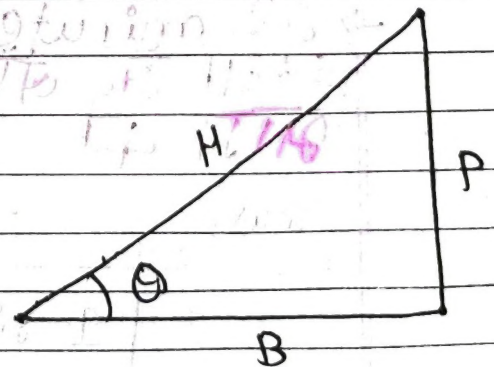
विशेषता इस Concept से है → यह ज्यामिती का
हम किसी भी दिए गए angle का
trigo. ratio निकाल
सकते हैं। Concept है।

★ कक्षा दसवीं में भी हमने Trigonometric
Ratios पढ़ा था लेकिन वो सिर्फ acute
angles के लिए applicable था।

$\sin \theta = \frac{P}{H}$ $\cot \theta = \frac{B}{P}$

$\cos \theta = \frac{B}{H}$ $\csc \theta = \frac{H}{P}$

$\tan \theta = \frac{P}{B}$ $\sec \theta = \frac{H}{B}$



Concept 2: Measurement of Angle

An angle is the amount of rotation of a revolving line with respect to a fixed line.

There are three system of measuring an angle :—

Sexagesimal System :—

→ इस व्यवस्था में एक right angle को 90 बराबर भागों में बाँटा गया है और एक भाग को कहाँ "degrees".

→ 1° symbol use करें है One degree को denote करने के लिए।

→ हर degree को further 60 बराबर हिस्सों में बाँटा गया और एक भाग को कहाँ "minutes".

→ हर minute को further 60 बराबर हिस्सों में बाँटा गया और एक भाग को कहाँ "seconds".

Thus,

$$1 \text{ right angle} = 90 \text{ degrees } (= 90^\circ)$$

$$1^\circ = 60 \text{ minutes } (60')$$

$$1' = 60 \text{ seconds } (60'')$$

ii) Centesimal System :-

- इस व्यवस्था में एक right angle को 100 बराबर भागों में बाँटा गया और एक भाग को कहा "grades".
- हर grade is further subdivided into 100 भागों में बाँटा गया और एक भाग को कहा "minutes".
- हर minute is further subdivided into 100 भागों में बाँटा गया और एक भाग को कहा "seconds".
- Symbols : —
- 1^g denotes a grade
 - $1'$ denotes a minute
 - $1''$ denotes a second

Thus,

1 right angle	= 100 grades (= 100^g)
1 grade	= 100 minutes (= $100'$)
1 minute	= 100 seconds (= $100''$)

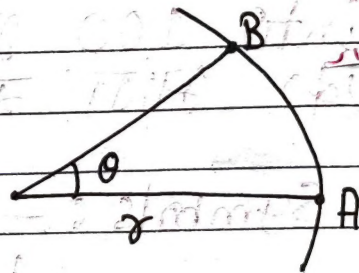
(iii) Circular System :-

→ इस व्यवस्था में unit of measurement is radian.

→ Radians :

One radian, written as 1^c , is the measure of an angle subtended at the centre of circle by an arc of length equal to radius of the circle.

$$\theta (\text{in radians}) = \frac{\text{arc AB}}{r}$$



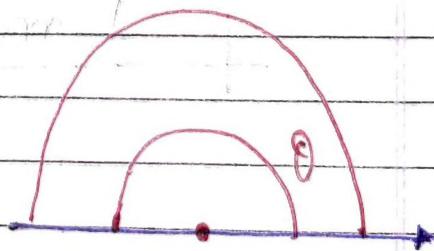
☆☆ Relation b/w degree and radians :-

Let θ be the angle subtended at centre by a semicircle $= 180^\circ$

From figure, $\theta = 180^\circ$

In radians :

$$\theta = \frac{\text{arc}}{\text{radius}} = \frac{\pi \cdot r}{r} = \pi$$



$$\Rightarrow 180^\circ \text{ degrees} = \pi \text{ radian}$$

Remember following:

Degree	Radian	Degree	Radian
		210°	$\frac{7\pi}{6}$
30°	$\frac{\pi}{6}$	225°	$\frac{5\pi}{4}$
45°	$\frac{\pi}{4}$	240°	$\frac{4\pi}{3}$
60°	$\frac{\pi}{3}$	270°	$\frac{3\pi}{2}$
90°	$\frac{\pi}{2}$	300°	$\frac{5\pi}{3}$
120°	$\frac{2\pi}{3}$	315°	$\frac{7\pi}{4}$
135°	$\frac{5\pi}{6}$	330°	$\frac{11\pi}{6}$
150°	$\frac{5\pi}{6}$	360°	2π
180°	π		

Concept 3: Measuring of angle in various quadrants

★ perpendicular lines XOX' and YOY' divide the plane in four parts.

Quadrant 1:

First Quadrant

(all points have $+X$ and $+Y$)

Q.2

Q.1

Quadrant 2:

Second Quadrant

(all points have $-X$ and $+Y$)

Q.3

Q.4

Quadrant 3:

Third Quadrant

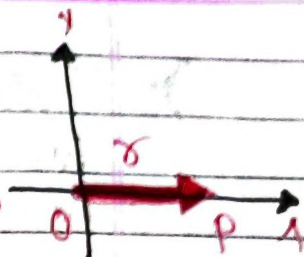
(all points have $-X$ and $-Y$)

Quadrant 4:

Fourth Quadrant

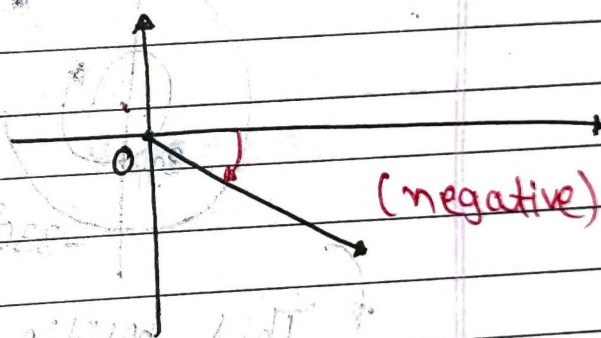
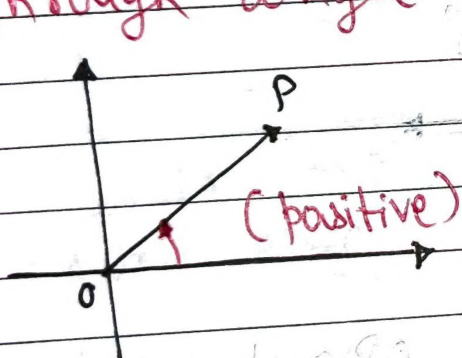
(all points have $+X$ and $-Y$)

➤ Every angle is represented by one position of a revolving ray OP of length " r ". The starting position for ray OP is taken along $+x$ axis.

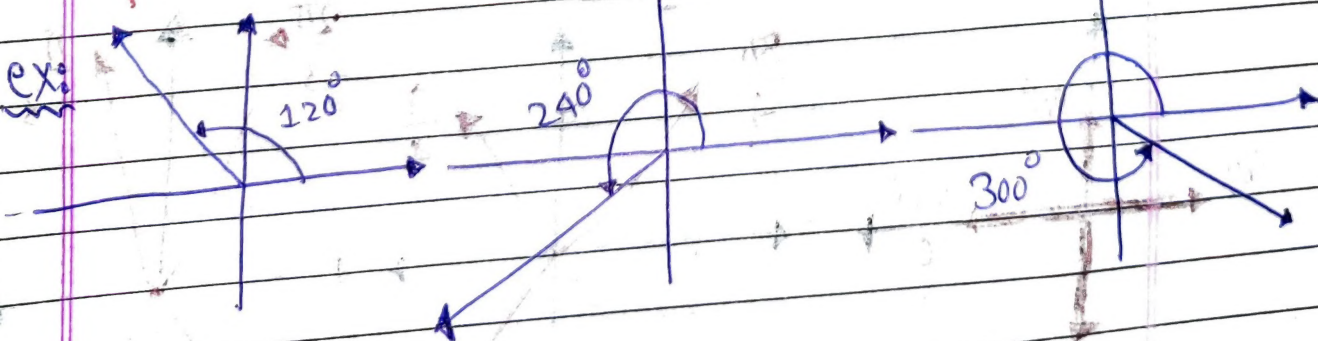


➤ If an angle α (alpha) is positive, OP rotates through angle α in anticlockwise direction (while),

If an angle α is negative, OP rotates through angle α in clockwise direction.

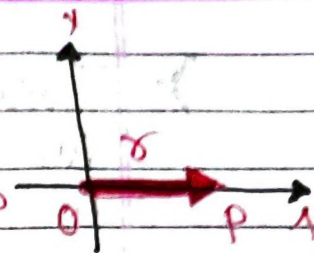


➤ An angle can lie in any of four quadrants according to the position of revolving ray for the angle.



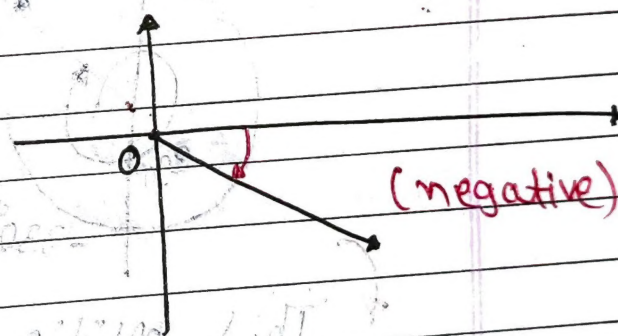
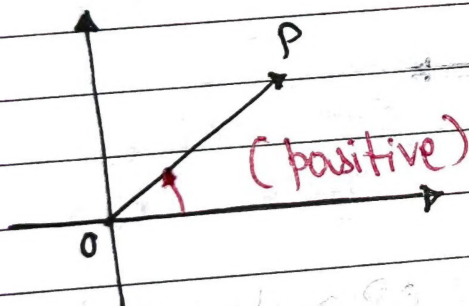
(P.T.O.)

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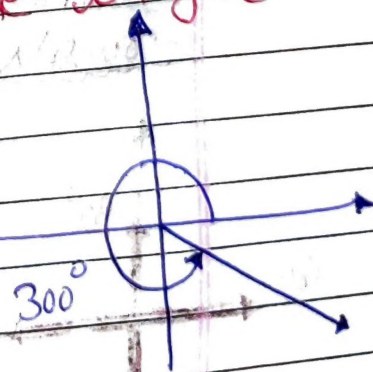
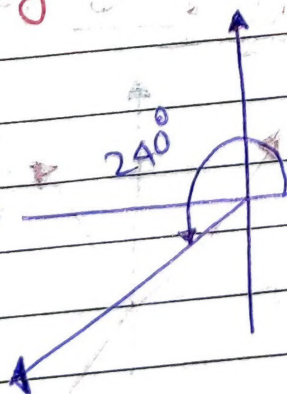
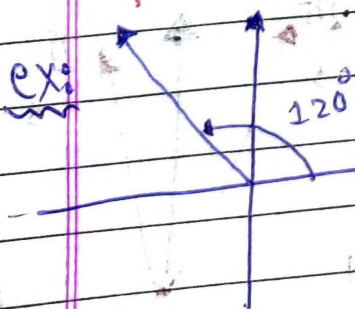


➤ If an angle α (alpha) is positive, OP rotates through angle α in anticlockwise direction (while),

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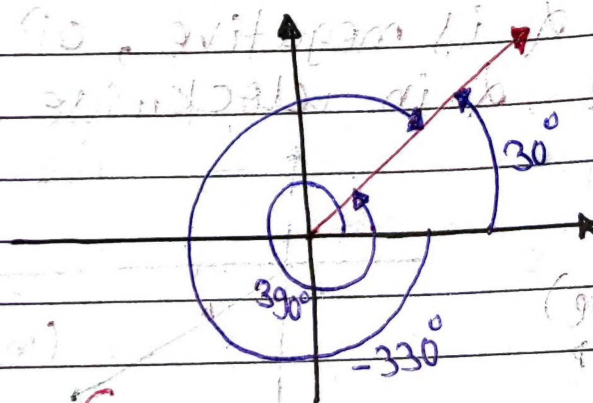
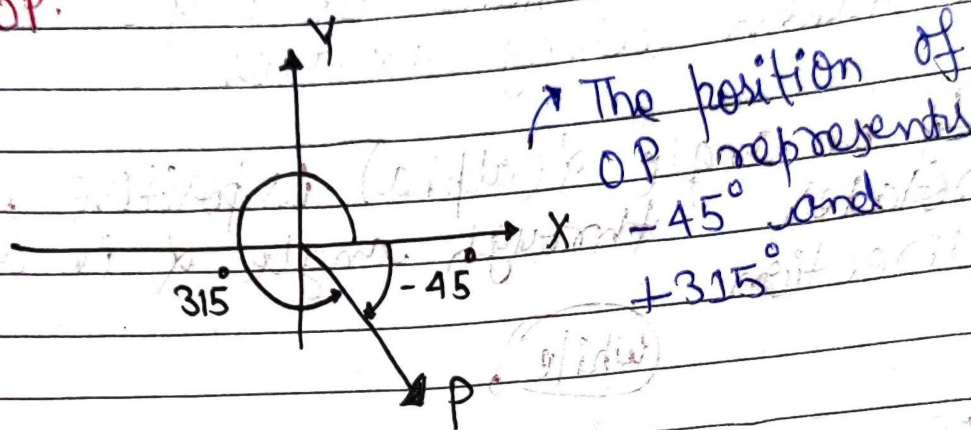


➤ An angle can lie in any of four quadrants according to the position of revolving ray for the angle.



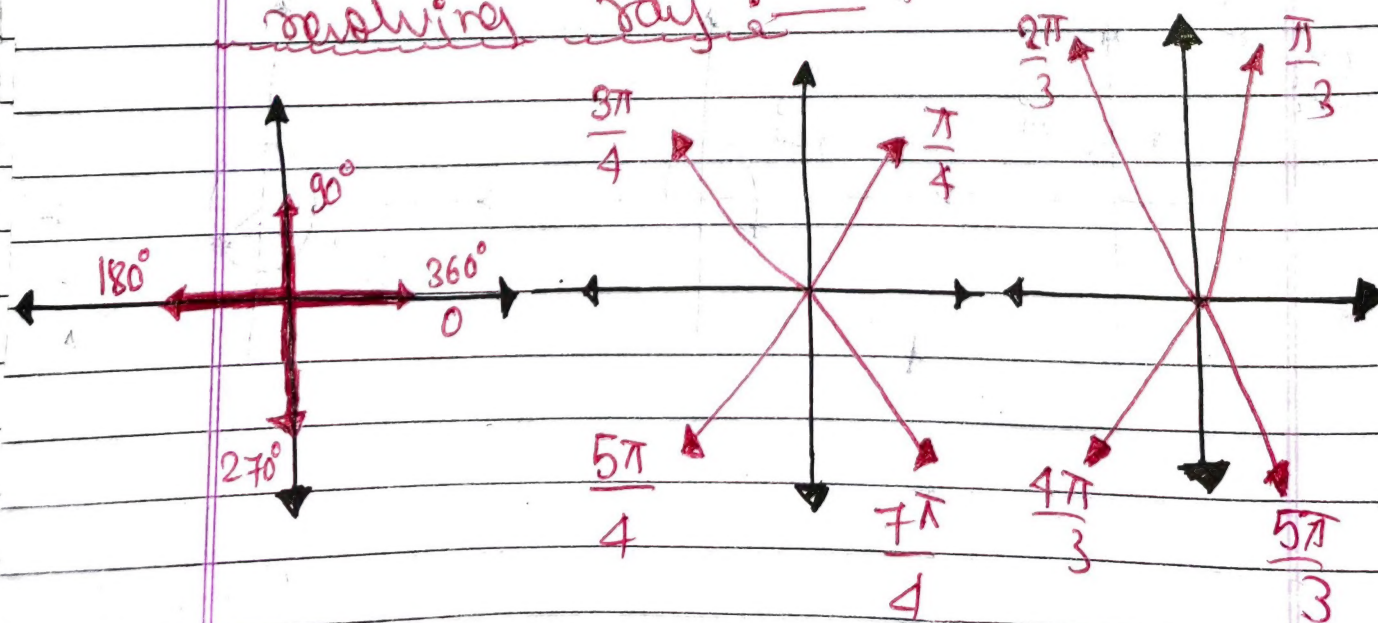
Two or more angles may correspond to some position of revolving ray OP.

ex:



This position of OP represents $+30^\circ$, $+390^\circ$ and -330° .

Remember following position of revolving ray:



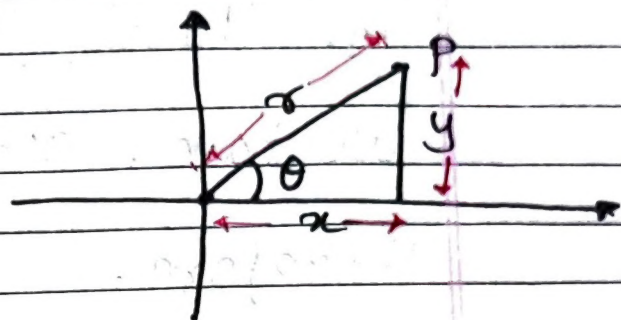
Concept : 4

Signs of Trigonometrical Functions

In First Quadrant

We have

$$x > 0, y > 0$$



Therefore,

$$\sin \theta = \frac{y}{r} > 0, \tan \theta = \frac{y}{x} > 0, \sec \theta = \frac{r}{x} > 0$$

$$\cos \theta = \frac{x}{r} > 0, \cot \theta = \frac{x}{y} > 0, \operatorname{cosec} \theta = \frac{r}{y} > 0$$

Thus, in the first quadrant all trigonometric functions are **positive**.

In Second Quadrant

We have $x < 0, y > 0$

Therefore,

$$\sin \theta = \frac{y}{r} > 0$$

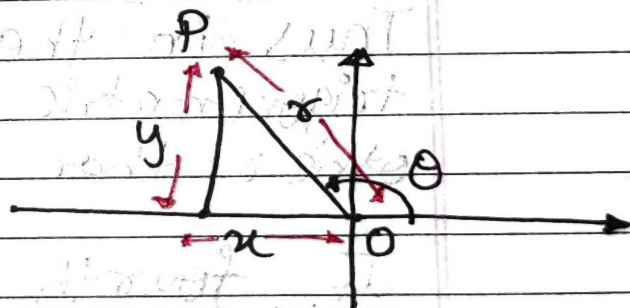
$$\cot \theta = \frac{x}{y} < 0$$

$$\cos \theta = \frac{x}{r} < 0$$

$$\sec \theta = \frac{r}{x} < 0$$

$$\tan \theta = \frac{y}{x} < 0$$

$$\operatorname{cosec} \theta = \frac{r}{y} > 0$$



Thus in the second quadrant all trigonometric function are negative other than cosecant.

In third quadrant

We have, $x < 0, y < 0$

Therefore,

$$\sin \theta = \frac{y}{r} < 0$$

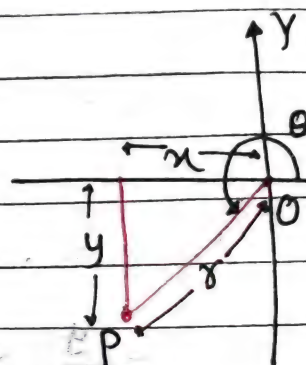
$$\cos \theta = \frac{x}{r} < 0$$

$$\tan \theta = \frac{y}{x} > 0$$

$$\cot \theta = \frac{x}{y} < 0$$

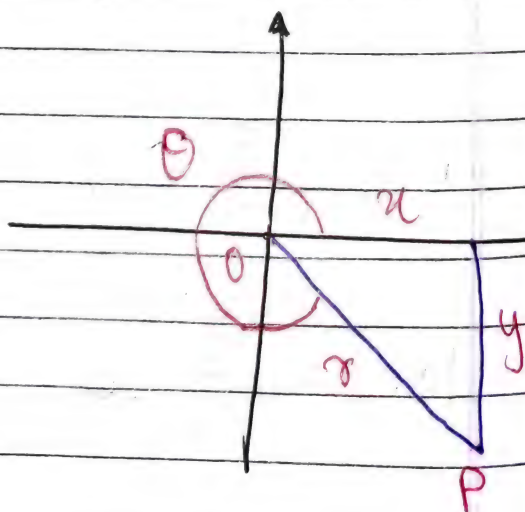
$$\sec \theta = \frac{r}{x} < 0$$

$$\csc \theta = \frac{r}{y} > 0$$



Thus in the third quadrant all trigonometric function are negative other than tangent & cotangent.

In fourth quadrant



we have $x > 0$, $y < 0$

$$\sin \theta = \frac{y}{r} < 0$$

$$\operatorname{cosec} \theta = \frac{r}{y} < 0$$

$$\cos \theta = \frac{x}{r} > 0$$

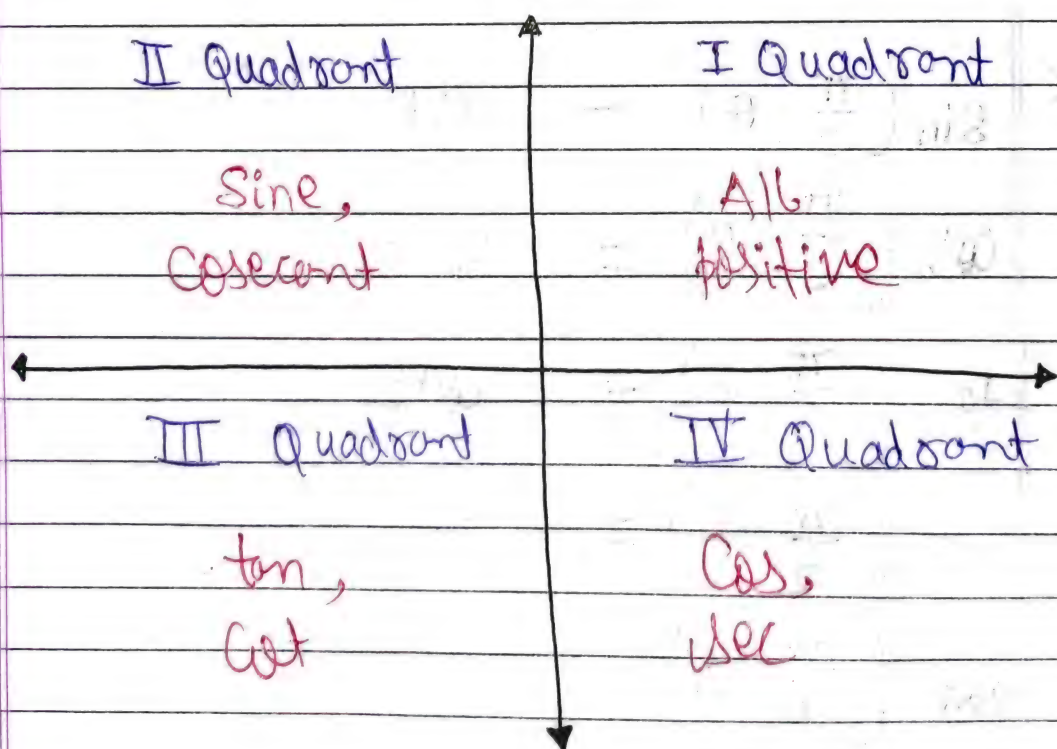
$$\sec \theta = \frac{r}{x} > 0$$

$$\tan \theta = \frac{y}{x} < 0$$

$$\cot \theta = \frac{x}{y} < 0$$

Thus in the fourth quadrant all trigonometric functions are negative other than cosine and secant.

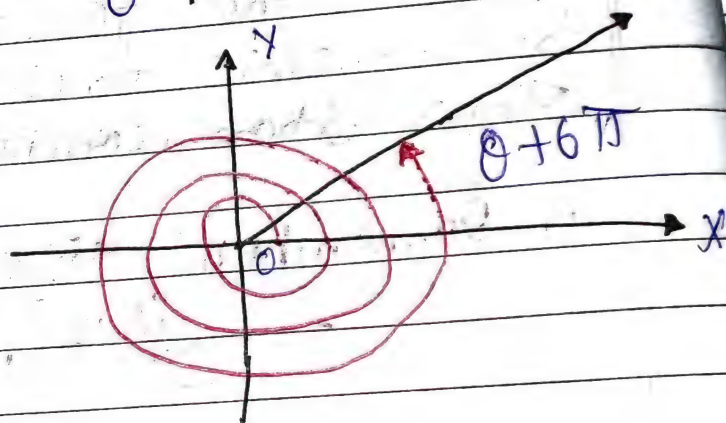
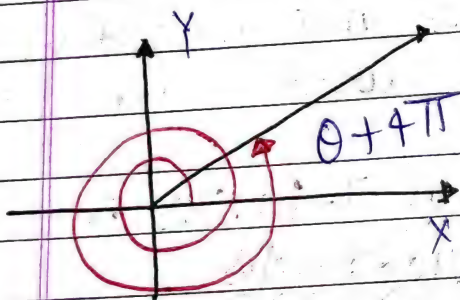
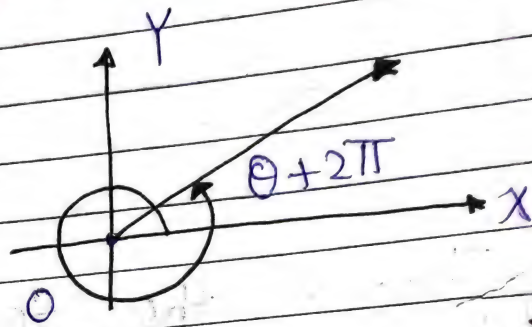
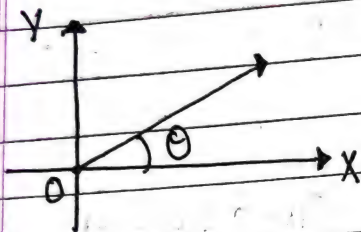
To Remember sign of T-Ratios in four quadrants.



Concept 5: Ratios of Allied angles

T-ratios for $2n\pi + \theta$:

Revolving ray assumes the same position for $\theta, 2\pi + \theta, 4\pi + \theta, 6\pi + \theta$.



$$\star \sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \operatorname{cosec} \theta$$

$$\operatorname{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

$$\star \sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta$$

$$\tan\left(\frac{\pi}{2} + \theta\right) = -\cot \theta$$

$$\cot\left(\frac{\pi}{2} + \theta\right) = -\tan \theta$$

$$\sec\left(\frac{\pi}{2} + \theta\right) = -\csc \theta$$

$$\csc\left(\frac{\pi}{2} + \theta\right) = \sec \theta$$

$$\star \sin(\pi - \theta) = \sin \theta$$

$$\cos(\pi - \theta) = -\cos \theta$$

$$\tan(\pi - \theta) = -\tan \theta$$

$$\cot(\pi - \theta) = -\cot \theta$$

$$\sec(\pi - \theta) = -\sec \theta$$

$$\csc(\pi - \theta) = \csc \theta$$

$$\star \sin(\pi + \theta) = -\sin \theta$$

$$\cos(\pi + \theta) = -\cos \theta$$

$$\tan(\pi + \theta) = \tan \theta$$

$$\cot(\pi + \theta) = \cot \theta$$

$$\sec(\pi + \theta) = -\sec \theta$$

$$\csc(\pi + \theta) = -\csc \theta$$



$$\sin\left(\frac{3\pi}{2} - \theta\right) = -\cos\theta$$

$$\cos\left(\frac{3\pi}{2} - \theta\right) = -\sin\theta$$

$$\tan\left(\frac{3\pi}{2} - \theta\right) = \cot\theta$$

$$\cot\left(\frac{3\pi}{2} - \theta\right) = \tan\theta$$

$$\sec\left(\frac{3\pi}{2} - \theta\right) = -\operatorname{cosec}\theta$$

$$\operatorname{cosec}\left(\frac{3\pi}{2} - \theta\right) = -\sec\theta$$



$$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos\theta$$

$$\cos\left(\frac{3\pi}{2} + \theta\right) = \sin\theta$$

$$\tan\left(\frac{3\pi}{2} + \theta\right) = -\cot\theta$$

$$\cot\left(\frac{3\pi}{2} + \theta\right) = -\tan\theta$$

$$\sec\left(\frac{3\pi}{2} + \theta\right) = \operatorname{cosec}\theta$$

$$\operatorname{cosec}\left(\frac{3\pi}{2} + \theta\right) = -\sec\theta$$

Trigonometric Ratios for sum & difference of angles

$$\star \sin(A+B) = \sin(A) \cdot \cos(B) + \cos(A) \cdot \sin(B)$$

$$\star \sin(A-B) = \sin(A) \cdot \cos(B) - \cos(A) \cdot \sin(B)$$

$$\star \cos(A+B) = \cos(A) \cdot \cos(B) - \sin(A) \cdot \sin(B)$$

$$\star \cos(A-B) = \cos(A) \cdot \cos(B) + \sin(A) \cdot \sin(B)$$

$$\star \tan(A+B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A) \cdot \tan(B)}$$

$$\star \tan(A-B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A) \cdot \tan(B)}$$

$$\star \cot(A+B) = \frac{\cot(A) \cdot \cot(B) - 1}{\cot(A) + \cot(B)}$$

$$\star \cot(A-B) = \frac{\cot(A) \cdot \cot(B) + 1}{\cot(B) - \cot(A)}$$

$$\rightarrow \tan(A+B+C) = \frac{\tan(A) + \tan(B) + \tan(C) - \tan(A) \cdot \tan(B) \cdot \tan(C)}{1 - \tan(A) \cdot \tan(B) - \tan(B) \cdot \tan(C) - \tan(C) \cdot \tan(A)}$$

$$\rightarrow \cot(A+B+C) = \frac{\cot(A) + \cot(B) + \cot(C) - \cot(A) \cdot \cot(B) \cdot \cot(C)}{1 - \cot(A) \cdot \cot(B) - \cot(B) \cdot \cot(C) - \cot(C) \cdot \cot(A)}$$

$$\rightarrow \sin(A+B) \cdot \sin(A-B) = \sin^2(A) - \sin^2(B) \\ = \cos^2(B) - \cos^2(A)$$

$$\rightarrow \cos(A+B) \cdot \cos(A-B) = \cos^2(A) - \sin^2(B) \\ = \cos^2(B) - \sin^2(A)$$

$$\rightarrow \sin(A+B+C) = \sin(A) \cdot \cos(B) \cdot \cos(C) + \cos(A) \cdot \sin(B) \cdot \cos(C) \\ + \cos(A) \cdot \cos(B) \cdot \sin(C) - \sin(A) \cdot \sin(B) \cdot \sin(C)$$

$$\rightarrow \cos(A+B+C) = \cos(A) \cdot \cos(B) \cdot \cos(C) - \sin(A) \cdot \sin(B) \cdot \cos(C) \\ - \sin(A) \cdot \cos(B) \cdot \sin(C) - \cos(A) \cdot \sin(B) \cdot \sin(C)$$

(P.T.O.)

Concept 9: Trigonometric Ratios of multiple and sub-multiple angles.

$$(i) \sin(2A) = 2 \cdot \sin(A) \cdot \cos(A)$$

$$(ii) \cos(2A) = \cos^2(A) - \sin^2(A)$$

$$(iii) \cos(2A) = 2 \cdot \cos^2(A) - 1$$

$$\textcircled{\text{or}}, 1 + \cos(2A) = 2 \cdot \cos^2(A)$$

$$(iv) \cos(2A) = 1 - 2 \sin^2(A)$$

$$\textcircled{\text{or}}, 1 - \cos(2A) = 2 \cdot \sin^2(A)$$

$$(v) \tan(2A) = \frac{2 \tan(A)}{1 - \tan^2(A)}$$

$$(vi) \sin(2A) = \frac{2 \tan(A)}{1 + \tan^2(A)}$$

$$(vii) \cos(2A) = \frac{1 - \tan^2(A)}{1 + \tan^2(A)}$$

$$(viii) \sin(3A) = 3 \sin(A) - 4 \sin^3(A)$$

$$(ix) \cos(3A) = 4 \cos^3(A) - 3 \cos(A)$$

$$(x) \tan(3A) = \frac{3 \tan(A) - \tan^3(A)}{1 - 3 \tan^2(A)}$$

Concept 10: Transformation formulae

★ Expressing Product of Trigonometric Functions as Sum or Difference

$$(i) 2 \cdot \sin(A) \cdot \cos(B) = \sin(A+B) + \sin(A-B)$$

$$(ii) 2 \cdot \cos(A) \cdot \sin(B) = \sin(A+B) - \sin(A-B)$$

$$(iii) 2 \cdot \cos(A) \cdot \cos(B) = \cos(A+B) + \cos(A-B)$$

$$(iv) 2 \cdot \sin(A) \cdot \sin(B) = \cos(A-B) - \cos(A+B)$$

★ Expressing Sum or Difference of two sines or two cosines as a product

$$\sin(C) + \sin(D) = 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$\sin(C) - \sin(D) = 2 \cdot \sin\left(\frac{C-D}{2}\right) \cdot \cos\left(\frac{C+D}{2}\right)$$

$$\cos(C) + \cos(D) = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$\cos(C) - \cos(D) = -2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$$

Practice Problems

Q. Find the value of:—

$$(i) \sin\left(\frac{25\pi}{3}\right) = \sin\left(8\pi + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\left[\because \sin(2n\pi + \theta) = \sin(\theta) \right]$$

$$(ii) \cos\left(\frac{41\pi}{4}\right) = \cos\left(10\pi + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$(iii) \tan\left(-\frac{16\pi}{3}\right) = \tan\left(-5\pi - \frac{\pi}{3}\right) = -\tan\left(5\pi + \frac{\pi}{3}\right) \\ = -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$$

$$(iv) \cot\left(\frac{29\pi}{4}\right) = \cot\left(7\pi + \frac{\pi}{4}\right) = \cot\left(\frac{\pi}{4}\right) = 1$$

$$(v) \sec\left(-\frac{19\pi}{3}\right) = \sec\left(-6\pi - \frac{\pi}{3}\right) = \sec\left(6\pi + \frac{\pi}{3}\right) \\ = \sec\left(\frac{\pi}{3}\right) = 2.$$

$$(vi) \operatorname{cosec}\left(-\frac{33\pi}{4}\right) = -\operatorname{cosec}\left(\frac{33\pi}{4}\right) \\ = -\operatorname{cosec}\left(8\pi + \frac{\pi}{4}\right) = -\operatorname{cosec}\left(\frac{\pi}{4}\right) \\ = -\sqrt{2}$$

Q. Find the value of:—

$$(i) \sin(765^\circ)$$

Ans $\because 180^\circ = \pi^\circ \Rightarrow 765^\circ = \left(\frac{\pi}{180^\circ} \times 765\right)^\circ$

$$= \left(\frac{17\pi}{4} \right)^c$$

$$\therefore \sin(765^\circ)$$

$$= \sin \left(4\pi + \frac{\pi}{4} \right)$$

$$= \sin \left(\frac{\pi}{4} \right) = \frac{1}{\sqrt{2}}$$

$$(ii) \operatorname{cosec}(-1110^\circ)$$

Ans: $\because 180^\circ = \pi^c$

$$\Rightarrow 1110^\circ = \left(\frac{\pi}{180} \times 1110 \right)^c = \left(\frac{37\pi}{6} \right)^c$$

$$\therefore \operatorname{cosec}(-1110^\circ) = -\operatorname{cosec}(1110^\circ)$$

$$= -\operatorname{cosec} \left(\frac{37\pi}{6} \right)$$

$$= -\operatorname{cosec} \left(6\pi + \frac{\pi}{6} \right)$$

$$= -\operatorname{cosec} \left(\frac{\pi}{6} \right)$$

$$= -2$$

(P.T.O.)

(iii) $\cot(-600^\circ)$

Ans:- $\because 180^\circ = \pi^c$

$$\Rightarrow 600^\circ = \left(\frac{\pi}{180} \times 600\right)^c = \left(\frac{10\pi}{3}\right)^c$$

$$\therefore \cot(-600^\circ) = -\cot(600^\circ)$$

$$= -\cot\left(\frac{10\pi}{3}\right)$$

$$= -\cot\left(3\pi + \frac{\pi}{3}\right) = -\cot\left(\frac{\pi}{3}\right)$$

$$= -\frac{1}{\sqrt{3}}$$

Q. Find the value of:—

(i) $\cos(15\pi) = \cos(14\pi + \pi)$
 $= \cos(\pi) = -1$

(ii) $\sin(16\pi) = \sin(16\pi + 0)$
 $= \sin 0^\circ = 0$

(iii) $\cos(-\pi) = \cos(\pi) = -1$

(iv) $\sin(5\pi) = \sin(4\pi + \pi)$
 $= \sin(\pi) = 0$

(v) $\tan\left(\frac{5\pi}{4}\right) = \because \frac{5\pi}{4} = 225^\circ \rightarrow 180^\circ + 45^\circ$
 $\therefore = \tan\left(\pi + \frac{\pi}{4}\right)$
 $= \tan\left(\frac{\pi}{4}\right) = 1$

Q. Find the value of :-

$$(i) \sin\left(\frac{31\pi}{3}\right) = \sin\left(10\pi + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$(ii) \cos\left(\frac{17\pi}{2}\right) = \cos\left(8\pi + \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

$$(iii) \tan\left(-\frac{25\pi}{3}\right) = -\tan\left(\frac{25\pi}{3}\right) \\ = -\tan\left(8\pi + \frac{\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$$

$$(iv) \cot\left(\frac{13\pi}{4}\right) = \cot\left(3\pi + \frac{\pi}{4}\right) \\ = \cot\left(\frac{\pi}{4}\right) = 1$$

$$(v) \sec\left(-\frac{25\pi}{3}\right) = \sec\left(\frac{25\pi}{3}\right) \\ = \sec\left(8\pi + \frac{\pi}{3}\right) = \sec\left(\frac{\pi}{3}\right) = 2$$

$$(vi) \operatorname{cosec}\left(-\frac{41\pi}{4}\right) = -\operatorname{cosec}\left(\frac{41\pi}{4}\right) \\ = -\operatorname{cosec}\left(10\pi + \frac{\pi}{4}\right) = -\operatorname{cosec}\left(\frac{\pi}{4}\right) = -\sqrt{2}$$

Q Find the value of:—

$$(i) \sin(405^\circ) = \text{?}$$

$\because 180^\circ = \pi^c$

$$\Rightarrow 405^\circ = \left(\frac{\pi}{180} \times 405 \right)^c$$

$$\Rightarrow \frac{9\pi}{4} = 2\pi + \frac{\pi}{4}$$

$$\Rightarrow \sin\left(2\pi + \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$(ii) \sec(-1470^\circ) =$$

$$\because 180^\circ = \pi^c$$

$$\Rightarrow -1470^\circ = -\left(\frac{\pi}{180} \times 1470 \right)^c$$

$$\Rightarrow \sec\left(-\frac{49\pi}{6}\right) = \sec\left(\frac{49\pi}{6}\right)$$

$$= \sec\left(8\pi + \frac{\pi}{6}\right) = \sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$$

$$(iii) \tan(-300^\circ) =$$

$$\because 180^\circ = \pi^c$$

$$\Rightarrow -300^\circ = -\left(\frac{\pi}{180} \times 300 \right)^c = -\frac{5\pi}{3}$$

$$\Rightarrow \tan\left(-\frac{5\pi}{3}\right) = -\tan\left(\frac{5\pi}{3}\right) = -\tan\left(2\pi - \frac{\pi}{3}\right)$$

$$\Rightarrow +\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

(iv) $\cot(585^\circ)$

$\because 180^\circ = \pi^c$ $\frac{360}{180} = 2$

Ans: $\Rightarrow 585^\circ = \left(\frac{\pi}{\frac{180}{36}} \times 585 \right)^c = \frac{13\pi}{4}$

$\Rightarrow \cot\left(\frac{13\pi}{4}\right) = \cot\left(3\pi + \frac{\pi}{4}\right) = \cot\left(\frac{\pi}{4}\right)$
 $= \cot\left(\frac{\pi}{4}\right) = 1$

(v) $\operatorname{cosec}(-750^\circ)$

Ans: $\because 180^\circ = \pi^c$

$\Rightarrow -750^\circ = -\left(\frac{\pi}{180} \times 750\right)^c = -\left(\frac{5\pi}{2}\right)^c$

$\Rightarrow -\operatorname{cosec}\left(\frac{5\pi}{2}\right)^c = -\operatorname{cosec}\left(4\pi + \frac{3\pi}{2}\right)$

$\Rightarrow = \operatorname{cosec}\left(\frac{3\pi}{2}\right) = -\operatorname{cosec}\left(\frac{\pi}{2}\right)$

$= -2$

(vi) $\cos(-2220^\circ)$

Ans: $\Rightarrow \cos\left(\frac{\pi}{180} \times 2220\right)^c = \cos\left(\frac{37\pi}{3}\right)^c$
 $= \cos\left(12\pi + \frac{\pi}{3}\right)^c$

$$= \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

Q. Find the value of $\sin^2\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{3}\right) - \tan^2\left(\frac{\pi}{4}\right)$.

Ans:
$$= \sin^2\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{3}\right) - \tan^2\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2 = \frac{2}{4} - 1 = -\frac{1}{2}$$

Q. Find the value of $\tan^2\left(\frac{\pi}{3}\right) + 2 \cdot \cos^2\left(\frac{\pi}{4}\right) + 3 \cdot \sec^2\left(\frac{\pi}{6}\right) + 4 \cdot \cot^2\left(\frac{\pi}{2}\right)$

Ans:
$$= (\sqrt{3})^2 + 2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 + 3 \cdot \left(\frac{2}{\sqrt{3}}\right)^2 + 4 \cdot 0$$

$$= 3 + 1 + \frac{3}{1} + 0$$

$$= 4 + \frac{3 \times 3}{4}$$

$$= (\sqrt{3})^2 + 2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 + 3 \cdot \left(\frac{2}{\sqrt{3}}\right)^2 + 4 \cdot (0)^2$$

$$= 3 + 1 + 4 + 0 = 8$$

Q. find the value of :-

$$4 \cdot \sin\left(\frac{\pi}{6}\right) \cdot \sin^2\left(\frac{\pi}{3}\right) + 3 \cdot \cos\left(\frac{\pi}{3}\right) \cdot \tan\left(\frac{\pi}{4}\right) + \sec^2\left(\frac{\pi}{2}\right)$$

Ans:
$$= 4 \times \frac{1}{2} \times \frac{3}{4} + 3 \times \frac{1}{2} \times 1 + (1)^2$$

$$= 2 \times \frac{3}{4} + \frac{3}{2} + 1$$

$$= \frac{3}{2} + \frac{3}{2} + 1 = \frac{8}{2} = 4$$

Q. Find the value of :-

(i) $\cos(480^\circ)$

$$\begin{aligned} \text{Ans:} &= \cos(360^\circ + 120^\circ) \\ &= \cos(120^\circ) \\ &= \cos(180^\circ - 60^\circ) \\ &= -\cos(60^\circ) \\ &= -\frac{1}{2} \end{aligned}$$

(ii) $\sin(1230^\circ)$

$$\begin{aligned} \text{Ans:} &= \sin(3 \times 360^\circ + 150^\circ) \\ &= \sin(150^\circ) \\ &= \sin(180^\circ - 30^\circ) \\ &= \sin(30^\circ) = \frac{1}{2} \end{aligned}$$

(iii) $\cot(-135^\circ)$

$$\begin{aligned} \text{Ans:} &= -\cot(135^\circ) \\ &= -\cot(90^\circ + 45^\circ) \\ &= \tan 45^\circ = 1 \end{aligned}$$

(iv) $\operatorname{cosec}(-1440^\circ)$

$$\begin{aligned} \text{Ans:} &= -\operatorname{cosec}(1440^\circ) \\ &= -\operatorname{cosec}(4 \times 360^\circ - 30^\circ) \\ &= \operatorname{cosec}(30^\circ) \\ &= 2 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad \cos(-870^\circ) &= \cos(870^\circ) \\
 &= \cos(2 \times 360^\circ + 150^\circ) \\
 &= \cos(150^\circ) \\
 &= \cos(180^\circ - 30^\circ) \\
 &= -\cos(30^\circ) = -\frac{\sqrt{3}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad \tan(330^\circ) &= \tan(360^\circ - 30^\circ) \\
 &= -\tan 30^\circ = -\frac{1}{\sqrt{3}}
 \end{aligned}$$

Q. Find the values of all trigonometric functions of :-

(i) 120° :-

$$\sin(120^\circ) = \sin(180^\circ - 60^\circ) = \sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(120^\circ) = \cos(180^\circ - 60^\circ) = -\cos(60^\circ) = -\frac{1}{2}$$

$$\tan(120^\circ) = \frac{\sin 120^\circ}{\cos 120^\circ} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$

$$\operatorname{cosec}(120^\circ) = \frac{1}{\sin(120^\circ)} = \frac{2}{\sqrt{3}}$$

$$\sec(120^\circ) = \frac{1}{\cos(120^\circ)} = -2$$

$$\cot(120^\circ) = \frac{1}{\tan(120^\circ)} = -\frac{1}{\sqrt{3}}$$

(ii) 150°

$$\sin(150^\circ) = \sin(180^\circ - 30^\circ) = \sin(30^\circ) = \frac{1}{2}$$

$$\cos(150^\circ) = \cos(180^\circ - 30^\circ) = -\cos(30^\circ) = -\frac{\sqrt{3}}{2}$$

$$\tan(150^\circ) = \frac{\sin(150^\circ)}{\cos(150^\circ)} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$\cot(150^\circ) = \frac{1}{\tan(150^\circ)} = -\sqrt{3}$$

$$\sec(150^\circ) = \frac{1}{\cos(150^\circ)} = -\frac{2}{\sqrt{3}}$$

$$\csc(150^\circ) = \frac{1}{\sin(150^\circ)} = 2$$

Q Show that :-

$$\sin(105^\circ) + \cos(105^\circ) = \frac{1}{2}$$

$$\begin{aligned} \text{Ans: } \text{L.H.S.} &= \sin(105^\circ) + \cos(105^\circ) \\ &= \sin(60^\circ + 45^\circ) + \cos(60^\circ + 45^\circ) \end{aligned}$$

$$= (\sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \cdot \sin 45^\circ) + (\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ)$$

$$= \left\{ \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \right) + \left(\frac{1}{2} \times \frac{1}{\sqrt{2}} \right) \right\} +$$

$$\left\{ \left(\frac{1}{2} \times \frac{1}{\sqrt{2}} \right) - \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \right) \right\}$$

$$= \left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} \right) = \frac{1}{\sqrt{2}}$$

*** Q Calculate the value of :-

(i.) $\sin 15^\circ$:-

$$\begin{aligned} &= \sin (45^\circ - 30^\circ) \\ &= \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ \\ &= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \right) - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2} \right) \end{aligned}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

(ii.) $\cos 15^\circ$:-

$$\begin{aligned} &= \cos (45^\circ - 30^\circ) \\ &= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ \\ &= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{2} \right) \end{aligned}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

(iii.) $\tan 15^\circ$:-

$$\begin{aligned} &= \tan (45^\circ - 30^\circ) \\ &= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ} \end{aligned}$$

$$= \frac{1 - 1/\sqrt{3}}{1 + 1 \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

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$$(iv.) \sin(75^\circ)$$

$$= \sin(45^\circ + 30^\circ)$$

$$= \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \cdot \sin 30^\circ$$

$$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{2} \right)$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$(v.) \cos(75^\circ)$$

$$= \cos(45^\circ + 30^\circ)$$

$$= \cos(45^\circ) \cdot \cos(30^\circ) - \sin(45^\circ) \cdot \sin(30^\circ)$$

$$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \right) - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2} \right)$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$(vi.) \tan(75^\circ)$$

$$= \tan(45^\circ + 30^\circ)$$

$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

Q. Evaluate $\tan\left(\frac{13\pi}{12}\right)$

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Ans:

$$\begin{aligned}\tan\left(\frac{13\pi}{12}\right) &= \tan\left(\pi + \frac{\pi}{12}\right) \\&= \tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\&= \frac{\tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)} \\&= \frac{\sqrt{3} - 1}{1 + \sqrt{3}}\end{aligned}$$

$$= \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{(\sqrt{3}-1)^2}{3-1}$$

$$= \frac{3+1-2\sqrt{3}}{2} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

Q. Prove that : —

$$(i) \sin 70^\circ \cdot \cos 10^\circ - \cos 70^\circ \cdot \sin 10^\circ = \frac{\sqrt{3}}{2}$$

Ans:

$$\because \sin \theta \cdot \cos \phi - \cos \theta \cdot \sin \phi = \sin(\theta - \phi)$$

$$\therefore = \sin(70 - 10^\circ)$$

$$= \sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$(ii) \cos 50^\circ \cdot \cos 10^\circ - \sin 50^\circ \cdot \sin 10^\circ = \frac{1}{2}$$

Ans: $\because \cos \theta \cdot \cos \phi - \sin \theta \cdot \sin \phi = \cos(\theta + \phi)$
 $= \cos(50^\circ + 10^\circ) = \cos 60^\circ = \frac{1}{2}$

$$(iii) \cos 80^\circ \cdot \cos 20^\circ + \sin 80^\circ \cdot \sin 20^\circ = \frac{1}{2}$$

Ans: $\because \cos \theta \cdot \cos \phi + \sin \theta \cdot \sin \phi = \cos(\theta - \phi)$
 $= \cos(80^\circ - 20^\circ) = \cos 60^\circ = \frac{1}{2}$

$$(iv) \sin 36^\circ \cdot \cos 9^\circ + \cos 36^\circ \cdot \sin 9^\circ = \frac{1}{\sqrt{2}}$$

Ans: $\because \sin \theta \cdot \cos \phi + \cos \theta \cdot \sin \phi = \sin(\theta + \phi)$
 $= \sin(36^\circ + 9^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$

Q. Prove that :-

$$\sin(40^\circ + \theta) \cdot \cos(10^\circ + \theta) - \cos(40^\circ + \theta) \cdot \sin(10^\circ + \theta) = 1/2$$

Ans: L.H.S. $= \sin(40^\circ + \theta) \cdot \cos(10^\circ + \theta) - \cos(40^\circ + \theta) \cdot \sin(10^\circ + \theta)$
 $= \sin[(40^\circ + \theta) - (10^\circ + \theta)]$
 $= \sin(30^\circ)$
 $= \frac{1}{2}$

Q. Prove that: -

$$\cos \theta + \sin(270^\circ + \theta) - \sin(270^\circ - \theta) + \cos(180^\circ + \theta) = 0$$

Ans:
$$= \cos \theta + \sin [180^\circ + (90^\circ + \theta)] - \sin [180^\circ + (90^\circ - \theta)] + \cos(180^\circ + \theta)$$

$$= \cos \theta - \sin(90^\circ + \theta) + \sin(90^\circ - \theta) - \cos \theta$$
$$= \cancel{\cos \theta} - \cancel{\cos \theta} + \cancel{\cos \theta} - \cancel{\cos \theta} = 0.$$

Q. Prove that: -

$$\frac{\cos(90^\circ + \theta) \cdot \sec(270^\circ + \theta) \cdot \sin(180^\circ + \theta)}{\csc(-\theta) \cos(270^\circ - \theta) \cdot \tan(180^\circ + \theta)} = \cos \theta.$$

Ans: L.H.S.

$$= \frac{\cos(90^\circ + \theta) \cdot \sec [180^\circ + (90^\circ + \theta)] \cdot \sin(180^\circ + \theta)}{-\csc(\theta) \cdot \cos [180^\circ + (90^\circ - \theta)] \cdot \tan(180^\circ + \theta)}$$

$$= \frac{\cos(90^\circ + \theta) \cdot \{-\sec(90^\circ + \theta)\} \cdot \{-\sin \theta\}}{-\csc(\theta) \cdot \{-\cos(90^\circ - \theta)\} \cdot \tan(\theta)}$$

$$= \frac{(-\sin \theta) \cdot \operatorname{cosec} \theta \cdot (-\sin \theta)}{(-\operatorname{cosec} \theta) \cdot (-\sin \theta) \cdot \tan \theta}$$

$$= \cos \theta$$

$$= \text{R.H.S.} \checkmark$$

Q. Prove that:-

$$\cos\left(\frac{3\pi}{2} + \theta\right) \cdot \cos(2\pi + \theta) \left[\cot\left(\frac{3\pi}{2} - \theta\right) + \cot(2\pi + \theta) \right] = 1$$

Ans:

$$= \cos\left(\frac{3\pi}{2} + \theta\right) \cdot \cos(2\pi + \theta) \left[\cot\left(\frac{3\pi}{2} - \theta\right) + \cot(2\pi + \theta) \right]$$

$$= \cos\left[\pi + \left(\frac{\pi}{2} + \theta\right)\right] \cdot \cos(2\pi + \theta) \left[\cot\left\{\pi + \left(\frac{\pi}{2} - \theta\right)\right\} + \cot(2\pi + \theta) \right]$$

$$= -\cos\left(\frac{\pi}{2} + \theta\right) \cdot \cos\theta \left[\cot\left(\frac{\pi}{2} - \theta\right) + \cot\theta \right]$$

$$= \sin\theta \cdot \cos\theta \left[\tan\theta + \cot\theta \right]$$

$$= (\sin\theta \cdot \cos\theta) \left[\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \right]$$

$$= (\sin\theta \cdot \cos\theta) \left[\frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta} \right] = 1$$

Q. Prove that:-

$$\sin\left(\frac{7\pi}{12}\right) \cdot \cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{7\pi}{12}\right) \cdot \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

Ans:

$$\left\{ \sin x \cdot \cos y - \cos x \cdot \sin y = \sin(x - y) \right\}$$

$$= \sin\left(\frac{7\pi}{12} - \frac{\pi}{4}\right)$$

$$= \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Q. Prove that :-

$$\sin\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{\pi}{12}\right) + \cos\left(\frac{\pi}{4}\right) \cdot \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}}{2}$$

Ans: $\left\{ \because \sin x \cdot \cos y + \cos x \cdot \sin y = \sin(x+y) \right\}$
$$= \sin\left(\frac{\pi}{4} + \frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Q. Prove that :-

$$\cos\left(\frac{2\pi}{3}\right) \cdot \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{2\pi}{3}\right) \cdot \sin\left(\frac{\pi}{4}\right) = -\frac{(\sqrt{3}+1)}{2\sqrt{2}}$$

Ans:

$$\begin{aligned} &= \cos\left(\pi - \frac{\pi}{3}\right) \cdot \cos\left(\frac{\pi}{4}\right) - \sin\left(\pi - \frac{\pi}{3}\right) \cdot \sin\left(\frac{\pi}{4}\right) \\ &= -\cos\left(\frac{\pi}{3}\right) \cdot \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right) \cdot \sin\left(\frac{\pi}{4}\right) \\ &= -\left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}\right) \\ &= -\left\{ \frac{(\sqrt{3}+1)}{2\sqrt{2}} \right\} \end{aligned}$$

Q. Evaluate :

(i) $\sin\left(\frac{\pi}{12}\right)$

Ans: $= \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$$= \sin\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right) \cdot \sin\left(\frac{\pi}{6}\right)$$

$$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)$$

$$= \frac{(\sqrt{3}-1)}{2\sqrt{2}}$$

(ii.) $\sin\left(\frac{5\pi}{12}\right)$

Ans:

$$= \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$= \sin\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right) \cdot \sin\left(\frac{\pi}{6}\right)$$

$$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)$$

$$= \frac{(\sqrt{3} + 1)}{2 \cdot \sqrt{2}}$$

Q. Prove that :-

$$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x + \tan y}{\tan x - \tan y}$$

Ans:

$$\text{L.H.S.} = \frac{\sin(x+y)}{\sin(x-y)}$$

$$= \frac{\sin x \cdot \cos y + \cos x \cdot \sin y}{\sin x \cdot \cos y - \cos x \cdot \sin y}$$

$$= \frac{\sin x \cdot \cos y}{\sin x \cdot \cos y - \cos x \cdot \sin y} + \frac{\cos x \cdot \sin y}{\sin x \cdot \cos y - \cos x \cdot \sin y}$$

$$= \frac{\sin x \cdot \cos y}{\sin x \cdot \cos y - \cos x \cdot \sin y} + \frac{\cos x \cdot \sin y}{\sin x \cdot \cos y - \cos x \cdot \sin y}$$

$$= \frac{\sin x \cdot \cos y}{\sin x \cdot \cos y - \cos x \cdot \sin y} + \frac{\cos x \cdot \sin y}{\sin x \cdot \cos y - \cos x \cdot \sin y}$$

$$= \frac{\tan x + \tan y}{\tan x - \tan y} = R.H.S.$$

Q. Prove that: —

$$\tan(56^\circ) = \frac{\cos(11^\circ) + \sin(11^\circ)}{\cos(11^\circ) - \sin(11^\circ)}$$

Ans:

$$L.H.S. = \tan(56^\circ)$$

$$= \tan(45^\circ + 11^\circ)$$

$$= \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \cdot \tan 11^\circ}$$

$$= \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$$

$$= \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ}$$

$$= \frac{1 + \frac{\sin 11^\circ}{\cos 11^\circ}}{1 - \frac{\sin 11^\circ}{\cos 11^\circ}} = \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = R.H.S.$$

Q. Prove that: —

$$\tan 70^\circ = \tan 20^\circ + 2 \tan 50^\circ$$

Ans:

$$L.H.S. = \tan 70^\circ$$

$$\Rightarrow \tan 70^\circ = \tan(20^\circ + 50^\circ)$$

$$\Rightarrow \tan 70^\circ = \frac{\tan 20^\circ + \tan 50^\circ}{1 - \tan 20^\circ \cdot \tan 50^\circ}$$

$$\Rightarrow \tan 70^\circ - \tan 20^\circ \cdot \tan 70^\circ \cdot \tan 50^\circ = \tan 20^\circ + \tan 50^\circ$$

$$\Rightarrow \tan 70^\circ - \tan (90 - 20^\circ) \cdot \tan 20^\circ \cdot \tan 50^\circ = \tan 20^\circ + \tan 50^\circ$$

$$\Rightarrow \tan 70^\circ - \cot 20^\circ \cdot \tan 20^\circ \cdot \tan 50^\circ = \tan 20^\circ + \tan 50^\circ$$

$$\Rightarrow \tan 70^\circ - \tan 50^\circ = \tan 20^\circ + \tan 50^\circ$$

$$\Rightarrow \tan 70^\circ = \tan 20^\circ + 2 \tan 50^\circ \text{ proved}$$

Q. Show that :-

$$\cot(2x) \cdot \cot(x) - \cot(3x) \cdot \cot(2x) - \cot(3x) \cdot \cot(x) = 1.$$

Ans:

we have,

$$\cot(3x) = \cot(2x + x)$$

$$\Rightarrow \cot(3x) = \frac{\cot(2x) \cdot \cot(x) - 1}{\cot(2x) + \cot(x)}$$

$$\Rightarrow \cot(3x) \cdot \cot(2x) + \cot(x) \cdot \cot(3x) = \cot(2x) \cdot \cot(x) - 1$$

$$\Rightarrow \cot(3x) \cdot \cot(2x) + \cot(x) \cdot \cot(3x) - \cot(2x) \cdot \cot(x) = -1$$

$$\Rightarrow \cos(2\pi) \cdot \cos(\pi) - \cos(3\pi) \cdot \cos(2\pi) - \cos(3\pi) \cdot \cos(\pi) \\ = 1.$$

Q. Find the value of :—

$$\begin{aligned} \text{(i.) } \cos(840^\circ) &= \cos(2 \times 360^\circ + 120^\circ) \\ &= \cos(120^\circ) \\ &= \cos(180^\circ - 60^\circ) \\ &= -\cos(60^\circ) = -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{(ii.) } \sin(870^\circ) &= \sin(2 \times 360^\circ + 150^\circ) \\ &= \sin(150^\circ) \\ &= \sin(180^\circ - 30^\circ) \\ &= \sin(30^\circ) \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{(iii.) } \tan(-120^\circ) &= -\tan(120^\circ) \\ &= -\tan(180^\circ - 60^\circ) \\ &= -\tan(-\tan(60^\circ)) \\ &= \tan(60^\circ) \\ &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{(iv.) } \sec(-420^\circ) &= \frac{1}{\cos(-420^\circ)} = \frac{1}{-\cos(420^\circ)} \\ &= \frac{1}{-\cos(360^\circ + 60^\circ)} = \frac{1}{-\cos(60^\circ)} = \frac{1}{-1/2} = -2. \end{aligned}$$

(v) $\operatorname{cosec}(-690^\circ)$

Ans: $= \frac{1}{\sin(-690^\circ)} = \frac{1}{-\sin(690^\circ)}$

$$= \frac{1}{-\sin(2 \times 360^\circ - 30^\circ)}$$

$$= \frac{1}{-(-\sin 30^\circ)} = \frac{1}{1/2} = 2$$

(vi) $\tan(225^\circ)$

Ans: $= \tan(180^\circ + 45^\circ)$
 $= \tan 45^\circ = 1$

(vii) $\cot(-315^\circ)$

Ans: $\Rightarrow \frac{1}{\tan(-315^\circ)} = \frac{1}{-\tan(315^\circ)}$

$$= \frac{1}{-\tan(360^\circ - 45^\circ)} = \frac{1}{+\tan 45^\circ} = +1$$

(viii) $\sin(-1230^\circ)$

Ans: $= -\sin(1230^\circ)$
 $= -\sin(3 \times 360^\circ + 150^\circ)$
 $= -\sin 150^\circ$
 $= -\sin(180^\circ - 30^\circ)$
 $= -\sin 30^\circ = -\frac{1}{2}$

Q. Prove that:-

(i) $\sin(80^\circ) \cdot \cos(20^\circ) - \cos(80^\circ) \cdot \sin(20^\circ) = \frac{\sqrt{3}}{2}$

Ans: $\left\{ \begin{array}{l} \because \sin A \cdot \cos B - \cos A \cdot \sin B = \sin(A-B) \\ \text{L.H.S.} \end{array} \right\}$
 $= \sin(80^\circ - 20^\circ)$
 $= \sin(60^\circ) = \frac{\sqrt{3}}{2}$

(ii) $\cos(45^\circ) \cdot \cos(15^\circ) - \sin(45^\circ) \cdot \sin(15^\circ) = \frac{1}{2}$

Ans: $\left\{ \begin{array}{l} \because \cos A \cdot \cos B - \sin A \cdot \sin B = \cos(A+B) \\ \text{L.H.S.} \end{array} \right\}$
 $= \cos(45^\circ + 15^\circ) = \cos(60^\circ) = \frac{1}{2}$

(iii) $\cos(75^\circ) \cdot \cos(15^\circ) + \sin(75^\circ) \cdot \sin(15^\circ) = \frac{1}{2}$

Ans: $\left\{ \begin{array}{l} \because \cos A \cdot \cos B + \sin A \cdot \sin B = \cos(A-B) \\ \text{L.H.S.} \end{array} \right\}$
 $= \cos(75^\circ - 15^\circ)$
 $= \cos(60^\circ) = \frac{1}{2}$

(iv) $\sin(40^\circ) \cdot \cos(20^\circ) + \cos(40^\circ) \cdot \sin(20^\circ) = \frac{\sqrt{3}}{2}$

Ans: $\left\{ \begin{array}{l} \because \sin A \cdot \cos B + \cos A \cdot \sin B = \sin(A+B) \\ \text{L.H.S.} \end{array} \right\}$
 $= \sin(40^\circ + 20^\circ)$
 $= \sin(60^\circ) = \frac{\sqrt{3}}{2}$

$$(v) \cos 130^\circ \cdot \cos 40^\circ + \sin 130^\circ \cdot \sin 40^\circ = 0$$

Ans: L.H.S.

$$= \cos (130^\circ - 40^\circ)$$

$$\left\{ \because \cos A \cdot \cos B + \sin A \cdot \sin B = \cos(A - B) \right\}$$

$$= \cos(90^\circ) = 0$$

Q. Prove that :-

$$\frac{\sin(50^\circ + \theta) \cdot \cos(20^\circ + \theta) - \cos(50^\circ + \theta) \cdot \sin(20^\circ + \theta)}{1} = \frac{1}{2}$$

Ans:

$$\left\{ \because \sin A \cdot \cos B - \cos A \cdot \sin B = \sin(A - B) \right\}$$

$$= \sin(50^\circ + \theta - 20^\circ - \theta)$$

$$= \sin(30^\circ)$$

$$= \frac{1}{2}$$

Q. Prove that: —

(i) $\cos(n+2)x \cdot \cos(n+1)x + \sin(n+2)x \cdot \sin(n+1)x$

$$= \cos(x)$$

Ans:

$$= \cos(n+2)x \cdot \cos(n+1)x + \sin(n+2)x \cdot \sin(n+1)x$$

$$= \left\{ \because \cos A \cdot \cos B + \sin A \cdot \sin B = \cos(A - B) \right\}$$

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$$\begin{aligned}
 &= \cos \{ (n+2) \cdot x - (n+1) \cdot x \} \\
 &= \cos \{ \cancel{nx} + 2x - \cancel{nx} - x \} \\
 &= \cos x = \text{R.H.S.}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad &\cos\left(\frac{\pi}{4} - x\right) \cdot \cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right) \cdot \sin\left(\frac{\pi}{4} - y\right) \\
 &= \sin(x+y)
 \end{aligned}$$

Ans:

$$\left\{ \because \cos A \cdot \cos B - \sin A \cdot \sin B = \cos(A+B) \right\}$$

$$= \cos\left(\frac{\pi}{4} - x + \frac{\pi}{4} - y\right)$$

$$= \cos\left(\frac{\pi}{2} - (x+y)\right) = \sin(x+y)$$

Q. Prove that :—

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan(x)}{1 - \tan(x)} \right)^2$$

Ans:

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\tan\left(\frac{\pi}{4}\right) + \tan(x)}{1 - \tan\left(\frac{\pi}{4}\right) \cdot \tan(x)} \\
 &= \frac{1 + \tan(x)}{1 - \tan(x)}
 \end{aligned}$$

$$= \frac{\frac{1 + \tan(x)}{1 - 1 \cdot \tan(x)}}{1 + 1 \cdot \tan(x)} = \left(\frac{1 + \tan(x)}{1 - \tan(x)} \right)^2$$

$$\therefore L.H.S. = R.H.S.$$

Q. Prove that :-

$$\sin(75^\circ) = \frac{(\sqrt{6} + \sqrt{2})}{4}$$

Ans:

$$\begin{aligned} L.H.S. &= \sin(75^\circ) \\ &= \sin(90^\circ - 15^\circ) \\ &= \sin(90^\circ) \cdot \cos(15^\circ) - \cos(90^\circ) \cdot \sin(15^\circ) \\ &= 1 \cdot \cos(15^\circ) - 0 \cdot \sin(15^\circ) \\ &= \cos(15^\circ) \end{aligned}$$

$$\begin{aligned} \cos(15^\circ) &= \cos(45^\circ - 30^\circ) \\ &= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \end{aligned}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}} \cdot 1$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

Q. $\frac{\cos(135^\circ) - \cos(120^\circ)}{\cos(135^\circ) + \cos(120^\circ)} = 3 - 2\sqrt{2}$

Ans: L.H.S. =

$$= \frac{\cos(180^\circ - 45^\circ) - \cos(180^\circ - 60^\circ)}{\cos(180^\circ - 45^\circ) + \cos(180^\circ - 60^\circ)}$$

$$= \frac{-\cos(45^\circ) - (-\cos(60^\circ))}{-\cos(45^\circ) + (-\cos(60^\circ))}$$

$$= \frac{\frac{1}{2} - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + \frac{1}{2}} = \frac{1 - \sqrt{2}}{\sqrt{2} + 1}$$

$$= \frac{1 - \sqrt{2}}{\sqrt{2} + 1} \times \frac{(-\sqrt{2} + 1)}{(-\sqrt{2} + 1)}$$

$$= \frac{\sqrt{2} + 1 + 2 - \sqrt{2}}{-2 + \sqrt{2} - \sqrt{2} + 1} = \frac{-2\sqrt{2} + 3}{-1}$$

$$= 3 - 2\sqrt{2}$$

(P.T.O.)

$$(iii) \tan(15^\circ) + \cot(15^\circ) = 4.$$

Ans: L.H.S.

$$= \tan(15^\circ) + \cot(15^\circ)$$

$$\left\{ \begin{array}{l} \therefore \text{we know,} \\ \sin(15^\circ) = \frac{\sqrt{3}-1}{2\sqrt{2}} \\ \cot(15^\circ) = \frac{\sqrt{3}+1}{2\sqrt{2}} \end{array} \right\}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} + \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{3+1-2\sqrt{3}+3+1+2\sqrt{3}}{3-1}$$

$$= \frac{8}{2} = 4 \Rightarrow \text{proved} \rightarrow$$

Q. Prove that:—

$$\cos(15^\circ) - \sin(15^\circ) = \frac{1}{\sqrt{2}} \quad)$$

Ans: L.H.S.

$$= \cos(15^\circ) - \sin(15^\circ)$$

$$\left\{ \begin{array}{l} \because \text{we know that,} \\ \sin(15^\circ) = \frac{\sqrt{3}-1}{2\sqrt{2}} ; \cos(15^\circ) = \frac{\sqrt{3}+1}{2\sqrt{2}} \end{array} \right\}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1 - \sqrt{3}+1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \quad \text{— R.H.S}$$

Q. Prove that:—

$$\cot(105^\circ) - \tan(105^\circ) = 2\sqrt{3}$$

Ans: L.H.S.

$$\begin{aligned} &= \cot(180^\circ - 75^\circ) - \tan(180^\circ - 75^\circ) \\ &= -\cot(75^\circ) - (-\tan(75^\circ)) \\ &= \tan(75^\circ) - \cot(75^\circ) \end{aligned}$$

we know,

$$\tan 75^\circ = \frac{-\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{-\sqrt{3}-1}{\sqrt{3}-1} - \frac{\sqrt{3}-1}{-\sqrt{3}-1}$$

$$= \frac{-\cancel{\sqrt{3}-1} + \cancel{\sqrt{3}-1}}{(\sqrt{3}-1)(1+\sqrt{3})}$$

~~$$= \frac{-2}{3-1} = -1$$~~

~~$$= \frac{-2}{\sqrt{3}+3-1-\sqrt{3}}$$~~

$$= \frac{\sqrt{3}-1}{-\sqrt{3}-1} - \frac{-\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2 - (-\sqrt{3}-1)^2}{(-\sqrt{3}-1)(\sqrt{3}-1)}$$

$$= \frac{-3+1-2\sqrt{3} - (3+1+2\sqrt{3})}{(-3+1-\sqrt{3}+\sqrt{3})}$$

$$= \frac{4 - 2\sqrt{3} - 4 - 2\sqrt{3}}{-2}$$

$$= \frac{-4\sqrt{3}}{-2} = 2\sqrt{3}$$

Q. prove that:-

$$\frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan(69^\circ) \cdot \tan(66^\circ)} = -1.$$

Ans:

$$\left\{ \because \frac{\tan(A) + \tan(B)}{1 - \tan(A) \cdot \tan(B)} = \tan(A+B) \right\}$$

$$= \tan(69^\circ + 66^\circ)$$

$$= \tan(135^\circ) = -1$$

Q. Prove that:-

$$\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \tan(54^\circ)$$

Ans:

$$\begin{aligned} & \text{L.H.S.} \\ &= \frac{\cancel{\cos 9^\circ} (1 + \tan 9^\circ)}{\cancel{\cos 9^\circ} (1 - \tan 9^\circ)} = \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \cdot \tan 9^\circ} \\ &= \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \cdot \tan 9^\circ} = \tan(45^\circ + 9^\circ) \\ &= \tan(54^\circ) = \text{R.H.S.} \end{aligned}$$

$$\left\{ \because \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = \tan(A+B) \right\}$$

Q. Prove that:—

$$\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan(37^\circ).$$

Ans: L.H.S.

$$= \frac{\cos 8^\circ (1 - \tan 8^\circ)}{\cos 8^\circ (1 + \tan 8^\circ)}$$

$$= \frac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 45^\circ \cdot \tan 8^\circ} = \tan(45^\circ - 8^\circ)$$

$$= \tan(37^\circ)$$

$$= R.H.S.$$

Q. Prove that:—

$$\frac{\cos(\pi + \theta) \cdot \cos(-\theta)}{\cos(\pi - \theta) \cdot \cos\left(\frac{\pi}{2} + \theta\right)} = -\cot(\theta)$$

$$L.H.S. = \frac{-\cos(\theta) \cdot \cos(\theta)}{\cos \theta \cdot -\sin \theta} = -\cot \theta.$$

Q. Prove that:—

$$\frac{\cos \theta}{\sin(90^\circ + \theta)} + \frac{\sin(-\theta)}{\sin(180^\circ + \theta)} - \frac{\tan(90^\circ + \theta)}{\cot \theta} = 3$$

Ans: L.H.S.

$$= \frac{\cancel{\cos(\theta)}}{\cancel{\cos(\theta)}} + \frac{\cancel{-\sin(\theta)}}{\cancel{-\sin(\theta)}} + \frac{\cancel{\cot(\theta)}}{\cancel{\cot(\theta)}}$$

$$= 1 + 1 + 1 = 3 = \text{R.H.S.}$$

Q. Prove that:—

$$\frac{\sin(180^\circ + \theta) \cdot \cos(90^\circ + \theta) \cdot \tan(270^\circ - \theta) \cdot \cot(360^\circ - \theta)}{\sin(360^\circ - \theta) \cdot \cos(360^\circ + \theta) \cdot \csc(-\theta) \cdot \sin(270^\circ + \theta)} = 1$$

Ans: L.H.S.

$$= \frac{(\cancel{-\sin(\theta)}) \times (\cancel{-\sin(\theta)}) \times \cot(\theta) \times (\cancel{\cot(\theta)})}{(\cancel{-\sin(\theta)}) \times \cos(\theta) \times \cancel{\csc(\theta)} \times \cancel{\cos(\theta)}}$$

$$= \frac{\cancel{\sin^2 \theta} \times \cos^2 \theta}{\cancel{\sin^2 \theta} \times \cancel{\csc^2 \theta}} = 1 = \text{R.H.S.}$$

Q. Prove that:—

(i) $\cos\left(\frac{\pi}{3} + x\right) = \frac{1}{2} \cdot (\cos(x) - \sqrt{3} \cdot \sin(x))$

Ans: $\left\{ \because \cos(A+B) = \cos(A) \cdot \cos(B) - \sin(A) \cdot \sin(B) \right\}$

$$= \frac{1}{2} \cdot \cos(x) - \frac{\sqrt{3}}{2} \cdot \sin(x)$$

$$= \frac{1}{2} (\cos(x) - \sqrt{3} \cdot \sin(x))$$

$$= \text{R.H.S.}$$

$$(ii) \sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right) = \sqrt{2} \cdot \cos(x)$$

Ans: $\left\{ \begin{array}{l} \because \sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B \\ \sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B \end{array} \right\}$

L.H.S.

$$= \sin\left(\frac{\pi}{4}\right) \cdot \cos(x) + \cos\left(\frac{\pi}{4}\right) \cdot \sin(x) + \sin\left(\frac{\pi}{4}\right) \cdot \cos(x) - \cos\left(\frac{\pi}{4}\right) \cdot \sin(x)$$

$$= 2 \cdot \sin\left(\frac{\pi}{4}\right) \cdot \cos(x)$$

$$= 2 \times \frac{1}{\sqrt{2}} \times \cos(x)$$

$$= \frac{2 \times \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} \times \cos(x) = \sqrt{2} \cdot \cos(x) = R.H.S.$$

$$(iii) \frac{1}{\sqrt{2}} \cdot \cos\left(\frac{\pi}{4} + x\right) = \frac{1}{2} (\cos(x) - \sin(x))$$

Ans: L.H.S. $\left\{ \because \cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B \right\}$

$$= \frac{1}{\sqrt{2}} \left(\cos\frac{\pi}{4} \cdot \cos(x) - \sin\left(\frac{\pi}{4}\right) \cdot \sin(x) \right)$$

$$= \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} \cdot \cos(x) - \frac{1}{\sqrt{2}} \cdot \sin(x) \right)$$

$$= \frac{1}{\sqrt{2} \times \sqrt{2}} [\cos(x) - \sin(x)]$$

$$= \frac{1}{2} (\cos(x) - \sin(x))$$

$$(iv) \cos(x) + \cos\left(\frac{2\pi}{3} + x\right) + \cos\left(\frac{2\pi}{3} - x\right) = 0$$

Ans. $\because \cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

I.H.S.

$$= \cos(x) + \cos\left(\frac{2\pi}{3}\right) \cdot \cos(x) - \sin\left(\frac{2\pi}{3}\right) \cdot \sin(x)$$

$$+ \cos\left(\frac{2\pi}{3}\right) \cdot \cos(x) + \sin\left(\frac{2\pi}{3}\right) \cdot \sin(x)$$

$$= \cos(x) + 2 \cdot \cos\left(\frac{2\pi}{3}\right) \cdot \cos(x)$$

$$= \cos(x) + 2 \cdot \cos\left(\pi - \frac{\pi}{3}\right) \cdot \cos(x)$$

$$= \cos(x) + 2 \times \left(-\frac{1}{2}\right) \cdot \cos(x) = 0$$

Q. Prove that :-

$$(i) 2 \cdot \sin\left(\frac{5\pi}{12}\right) \cdot \sin\left(\frac{\pi}{12}\right) = \frac{1}{2}$$

I.H.S. =

$$\left\{ \because 2 \cdot \sin(A) \cdot \sin(B) = \cos(A-B) - \cos(A+B) \right\}$$

$$= - \left(\cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) - \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right) \right)$$

$$= - \left(\cos\left(\frac{6\pi}{12}\right) - \cos\left(\frac{4\pi}{12}\right) \right)$$

$$= - \left(\cos\left(\frac{\pi}{2}\right) - \cos\left(\frac{\pi}{3}\right) \right)$$

$$= - \left(0 - \frac{1}{2} \right) = \frac{1}{2} = \text{R.H.S.}$$

$$(ii) 2 \cdot \cos\left(\frac{5\pi}{12}\right) \cdot \cos\left(\frac{\pi}{12}\right) = \frac{1}{2}$$

Ans: L.H.S.

$$= \left\{ \because 2 \cos A \cos B = \cos(A+B) + \cos(A-B) \right\}$$

$$= \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) + \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$$

$$= \cos\left(\frac{6\pi}{12}\right) + \cos\left(\frac{4\pi}{12}\right)$$

$$= \cos\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{3}\right)$$

$$= 0 + \frac{1}{2} = \frac{1}{2} = \text{R.H.S.}$$

Q. If $\sin(\theta) = \frac{15}{17}$ and $\cos(\phi) = \frac{12}{13}$,

where θ and ϕ both lie in the first quadrant, find the values of:-

(i) $\sin(\theta + \phi)$

(ii) $\cos(\theta - \phi)$

(iii) $\tan(\theta + \phi)$

(P.T.O.)

Ans:

$$\therefore \sin(\theta) = \frac{15}{17} \quad \& \quad \cos(\phi) = \frac{12}{13}$$

$\therefore \theta$ lies in the first quadrant, we have
 $\sin(\theta) > 0$, $\cos(\theta) > 0$ and $\tan(\theta) > 0$.

again,

$\therefore \phi$ lies in the first quadrant, we have
 $\sin(\phi) > 0$, $\cos(\phi) > 0$ and $\tan(\phi) > 0$.

Now,

$$\begin{aligned} \cos^2 \theta &= 1 - \sin^2 \theta \\ &= \left(1 - \frac{225}{289}\right) = \frac{64}{289} \end{aligned}$$

$$\cos \theta = + \sqrt{\frac{64}{289}} = \frac{8}{17}$$

And,

$$\begin{aligned} \sin^2 \phi &= (1 - \cos^2 \phi) \\ &= \left(1 - \frac{144}{169}\right) = \frac{25}{169} \end{aligned}$$

$$\Rightarrow \sin \phi = + \sqrt{\frac{25}{169}} = \frac{5}{13}$$

$$\therefore \tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{15}{17} \times \frac{17}{8} = \frac{15}{8} \quad (4)$$

$$\tan(\phi) = \frac{\sin(\phi)}{\cos(\phi)} = \frac{5}{13} \times \frac{13}{12} = \frac{5}{12}$$

(P.T.O.)

Now,

$$\begin{aligned}
 \text{(i.) } \sin(\theta + \phi) &= \sin \theta \cdot \cos \phi + \cos \theta \cdot \sin \phi \\
 &= \left(\frac{15}{17} \times \frac{12}{13} \right) + \left(\frac{8}{17} \times \frac{5}{13} \right) \\
 &= \left(\frac{180}{221} + \frac{40}{221} \right) \\
 &= \frac{220}{221}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii.) } \cos(\theta - \phi) &= \cos \theta \cdot \cos \phi + \sin \theta \cdot \sin \phi \\
 &= \left(\frac{8}{17} \times \frac{12}{13} \right) + \left(\frac{15}{17} \times \frac{5}{13} \right) \\
 &= \left(\frac{96}{221} + \frac{75}{221} \right) = \frac{171}{221}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii.) } \tan(\theta + \phi) &= \frac{\tan \theta + \tan \phi}{1 - \tan \theta \cdot \tan \phi} \\
 &\Rightarrow \frac{\left(\frac{15}{8} + \frac{5}{12} \right)}{\left\{ 1 - \left(\frac{15}{8} \times \frac{5}{12} \right) \right\}} = \frac{\frac{55}{24}}{1 - \frac{25}{32}} \\
 &\Rightarrow \left(\frac{55}{24} \times \frac{32}{7} \right) = \frac{220}{21}
 \end{aligned}$$

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Q. If $\cot(\alpha) = \frac{1}{2}$ and $\sec(\beta) = -\frac{5}{3}$, where $\alpha \in]\pi, \frac{3\pi}{2}[$ and $\beta \in]\frac{\pi}{2}, \pi[$, find the value of $\tan(\alpha + \beta)$.

Ans: Here, α lies in Quadrant III & therefore, $\tan(\alpha)$ is positive.

Now, $\cot(\alpha) = \frac{1}{2} \Rightarrow \tan(\alpha) = 2$

again,

β lies in Quadrant II and, therefore, $\sin(\beta)$ is positive and $\cos(\beta)$ is negative

Now, $\sec(\beta) = -\frac{5}{3} \Rightarrow \cos(\beta) = -\frac{3}{5}$

and,

$$\begin{aligned}\sin(\beta) &= +\sqrt{1 - \cos^2(\beta)} \\ &= +\sqrt{1 - \frac{9}{25}} = +\sqrt{\frac{16}{25}} = \frac{4}{5}\end{aligned}$$

$$\therefore \tan(\beta) = \frac{\sin(\beta)}{\cos(\beta)} = \frac{4}{5} \times \left(-\frac{5}{3}\right) = -\frac{4}{3}$$

$$\therefore \tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha) \cdot \tan(\beta)}$$

$$\Rightarrow \frac{2 - \frac{4}{3}}{1 - 2 \times \left(-\frac{4}{3}\right)} = \frac{\left(\frac{2}{3}\right)}{\left(\frac{11}{3}\right)}$$

$$\Rightarrow \frac{2}{3} \times \frac{3}{11} = \frac{2}{11} = \textcircled{0}$$

Q. If x and y are acute angles such that $\sin(x) = \frac{1}{\sqrt{5}}$ and $\sin(y) = \frac{1}{\sqrt{10}}$, prove that $(x+y) = \frac{\pi}{4}$.

Ans: given,

$$\sin(x) = \frac{1}{\sqrt{5}} \quad \& \quad \sin(y) = \frac{1}{\sqrt{10}}$$

$$\cos(x) = \sqrt{1 - \sin^2(x)}$$

$$= \sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2}$$

$$= \sqrt{\frac{5-1}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\cos(y) = \sqrt{1 - \sin^2(y)}$$

$$= \sqrt{1 - \left(\frac{1}{\sqrt{10}}\right)^2}$$

$$= \sqrt{\frac{10-1}{10}} = \frac{3}{\sqrt{10}}$$

$$\begin{aligned} \therefore \sin(x+y) &= \sin(x) \cdot \cos(y) + \cos(x) \cdot \sin(y) \\ &= \frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}} \\ &= \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{50}} = \frac{5}{\sqrt{50}} = \frac{5}{5\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$

$$\Rightarrow \sin(x+y) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x+y = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow x+y = \frac{\pi}{4} \quad \text{proved}$$

Q. If x and y are acute angles such that $\cos(x) = \frac{13}{14}$ and $\cos(y) = \frac{1}{7}$ prove that $(x-y) = -\frac{\pi}{3}$.

Ans: given,

$$\cos(x) = \frac{13}{14} \quad \text{and} \quad \cos(y) = \frac{1}{7}$$

$$\sin(x) = \sqrt{1 - \cos^2(x)}$$

(P.T.O.)

$$= \sqrt{1 - \left(\frac{13}{14}\right)^2} \Rightarrow \sqrt{\frac{196 - 169}{196}}$$

$$= \sqrt{\frac{27}{196}} = \frac{3\sqrt{3}}{14}$$

$$\sin(y) = \sqrt{1 - \cos^2(y)}$$

$$= \sqrt{1 - \left(\frac{1}{7}\right)^2} = \sqrt{\frac{49 - 1}{49}}$$

$$= \sqrt{\frac{48}{49}} = \frac{4\sqrt{3}}{7}$$

hence,

$$\cos(x-y) = \cos(x) \cdot \cos(y) + \sin(x) \cdot \sin(y)$$

$$= \frac{13}{14} \times \frac{1}{7} + \frac{3\sqrt{3}}{14} \times \frac{4\sqrt{3}}{7}$$

$$= \frac{13 + 36}{98} = \frac{49}{98}$$

$$\cos(x-y) = \frac{1}{2}$$

$$(x-y) = \frac{\pi}{3}$$

Q. Prove that:—

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$$\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = \sqrt{2} \cdot \cos(x)$$

Ans:

L.H.S.

$$= \left\{ \because \cos(A+B) + \cos(A-B) = 2 \cdot \cos A \cdot \cos B \right\}$$

$$= 2 \cdot \cos\left(\frac{\pi}{4}\right) \cdot \cos(x)$$

$$= \left(2 \times \frac{1}{\sqrt{2}} \cdot \cos(x)\right) = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \times \cos(x)$$

$$= \sqrt{2} \cdot \cos(x) = \text{R.H.S.}$$

Q. Prove that:—

$$\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \cdot \sin(x)$$

Ans:

$$\left\{ \because \cos(A+B) - \cos(A-B) = -2 \cdot \sin A \cdot \sin B \right\}$$

$$= -2 \cdot \sin\left(\frac{3\pi}{4}\right) \cdot \sin(x)$$

$$= -2 \cdot \sin\left(\pi - \frac{\pi}{4}\right) \cdot \sin(x)$$

$$= -2 \cdot \sin \frac{\pi}{4} \cdot \sin(x)$$

$$= \left(-2 \times \frac{1}{\sqrt{2}}\right) \cdot \sin(x)$$

$$= -\sqrt{2} \cdot \sin(x) = \text{R.H.S.}$$

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Q. Express each of the following as an algebraic sum of sines or cosines.

(i) $2 \cdot \sin(3x) \cdot \cos(2x)$

Ans: $\left\{ \because 2 \cdot \sin(A) \cdot \cos(B) = \sin(A+B) + \sin(A-B) \right\}$
 $= \sin(3x+2x) + \sin(3x-2x)$
 $= \sin(5x) + \sin(x)$

(ii) $2 \cdot \cos(4x) \cdot \cos(2x)$

Ans: $\left\{ \because 2 \cdot \cos(A) \cdot \cos(B) = \cos(A+B) + \cos(A-B) \right\}$
 $= \cos(4x+2x) + \cos(4x-2x)$
 $= \cos(6x) + \cos(2x)$

(iii) $2 \cdot \cos(6x) \cdot \cos(4x)$

Ans: $\left\{ \because 2 \cdot \cos(A) \cdot \cos(B) = \cos(A+B) + \cos(A-B) \right\}$
 $= \cos(6x+4x) + \cos(6x-4x)$
 $= \cos(10x) + \cos(2x)$

(iv) $2 \cdot \sin(3x) \cdot \sin(5x)$

Ans: $\left\{ \because 2 \sin A \cdot \sin B = \cos(A-B) - \cos(A+B) \right\}$
 $= 2 \cdot \sin(5x) \cdot \sin(3x)$

(iii) $\sin(7x) + \sin(3x)$

Ans: $\because \sin C + \sin D = 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

$$\Rightarrow 2 \cdot \sin\left(\frac{7x+3x}{2}\right) \cdot \cos\left(\frac{7x-3x}{2}\right)$$

$$\Rightarrow 2 \cdot \sin(5x) \cdot \cos(2x)$$

(iv) $\sin(5x) - \sin(3x)$

Ans: $\because \sin C - \sin D = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$$= 2 \cdot \cos\left(\frac{5x+3x}{2}\right) \cdot \sin\left(\frac{5x-3x}{2}\right)$$

$$= 2 \cdot \cos(4x) \cdot \sin(x)$$

Q. Prove that :-

$$\frac{\cos(6x) + \cos(4x)}{\sin(6x) - \sin(4x)} = \cot(x)$$

Ans: L.H.S.

$$= \frac{\cos(6x) + \cos(4x)}{\sin(6x) - \sin(4x)}$$

$$= \frac{2 \cdot \cos\left(\frac{6x+4x}{2}\right) \cdot \cos\left(\frac{6x-4x}{2}\right)}{\sin(6x) - \sin(4x)}$$

$$= \frac{2 \cdot \cos(5x) \cdot \cos(x)}{\sin(6x) - \sin(4x)}$$

$$\left\{ \begin{aligned} \therefore \cos C + \cos D &= 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \\ \sin C - \sin D &= 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \end{aligned} \right.$$

$$= \frac{2 \cdot \cos(5\pi) \cdot \cos(\pi)}{2 \cdot \cos(5\pi) \cdot \sin(\pi)} = \cot(\pi) = \text{R.H.S.}$$

Q. Prove that: —

$$\frac{\sin(3\pi) - \sin(\pi)}{\cos(\pi) - \cos(3\pi)} = \cot(2\pi)$$

Ans: L.H.S.

$$= \frac{\sin(3\pi) - \sin(\pi)}{\cos(\pi) - \cos(3\pi)}$$

$$= \left\{ \begin{aligned} \therefore \sin C - \sin D &= 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \\ \cos C - \cos D &= -2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \end{aligned} \right.$$

$$= \frac{2 \cdot \cos\left(\frac{3\pi+\pi}{2}\right) \cdot \sin\left(\frac{3\pi-\pi}{2}\right)}{-2 \cdot \sin\left(\frac{\pi+3\pi}{2}\right) \cdot \sin\left(\frac{\pi-3\pi}{2}\right)}$$

$$\Rightarrow \frac{\cos(2\pi) \cdot \sin(\pi)}{-\sin(2\pi) \cdot \sin(-\pi)} = \frac{\cos(2\pi) \cdot \sin(\pi)}{\sin(2\pi) \cdot \sin(\pi)}$$

$$= \cot(2\pi) = \text{R.H.S.}$$

Q. Prove that :-

$$\frac{\sin(3x) - \sin(x)}{\cos(2x)} = 2 \cdot \sin(x)$$

Ans:

L.H.S.

$$= \left\{ \because \sin(C) - \sin(D) = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \right\}$$

$$= \frac{2 \cdot \cos\left(\frac{3x+x}{2}\right) \cdot \sin\left(\frac{3x-x}{2}\right)}{\cos(2x)}$$

$$= \frac{2 \cdot \cos(2x) \cdot \sin(x)}{\cancel{\cos(2x)}} = 2 \cdot \sin(x)$$

$$= \cancel{\cos(2x)} = R.H.S.$$

Q. Prove that :-

$$\frac{\sin(5x) - 2 \cdot \sin(3x) + \sin(x)}{\cos(5x) - \cos(x)} = \cos(2x) - \cos(2x)$$

Ans:

L.H.S.

$$\left\{ \begin{aligned} \because \sin C + \sin D &= 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \\ \cos C - \cos D &= 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \end{aligned} \right\}$$

$$= \frac{(\sin(5x) + \sin(x)) - 2 \cdot \sin(3x)}{\cos(5x) - \cos(x)}$$

$$\begin{aligned}
 &= \frac{2 \cdot \sin\left(\frac{5\pi + \pi}{2}\right) \cdot \cos\left(\frac{5\pi - \pi}{2}\right) - 2 \sin(3\pi)}{-2 \cdot \sin\left(\frac{5\pi + \pi}{2}\right) \cdot \sin\left(\frac{5\pi - \pi}{2}\right)} \\
 &= \frac{2 \cdot \sin(3\pi) \cdot \cos(2\pi) - 2 \cdot \sin(3\pi)}{-2 \cdot \sin(3\pi) \cdot \sin(2\pi)} \\
 &= \frac{\cancel{2 \sin(3\pi)} [\cos(2\pi) - 1]}{\cancel{-2 \sin(3\pi)} \cdot \sin(2\pi)} \\
 &= \frac{1 - \cos(2\pi)}{\sin(2\pi)} = \frac{1}{\sin(2\pi)} - \frac{\cos(2\pi)}{\sin(2\pi)} \\
 &= \operatorname{cosec}(2\pi) - \cot(2\pi) \\
 &= \text{R.H.S.}
 \end{aligned}$$

Q. If $\frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)}$, prove that

$$\tan(A) \cdot \tan(B) \cdot \tan(C) + \tan(D) = 0.$$

Ans: L.H.S.

We have,

$$\frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)}$$

$$\Rightarrow \frac{\cos(A+B) + \cos(A-B)}{\cos(A+B) - \cos(A-B)} = \frac{\sin(C+D) + \sin(C-D)}{\sin(C+D) - \sin(C-D)}$$

$$\Rightarrow \frac{2 \cdot \cos(A) \cdot \cos(B)}{-2 \cdot \sin(A) \cdot \sin(B)} = \frac{2 \cdot \sin(C) \cdot \cos(D)}{2 \cdot \cos(C) \cdot \sin(D)}$$

$$\Rightarrow -\cot(A) \cdot \cot(B) = \tan(C) \cdot \cot(D)$$

$$\Rightarrow \tan A \cdot \tan B \cdot \tan C = -\tan D$$

$$\Rightarrow \tan A \cdot \tan B \cdot \tan C + \tan D = 0$$

Proved!

Q Prove that: —

$$\frac{\cos(4x) + \cos(3x) + \cos(2x)}{\sin(4x) + \sin(3x) + \sin(2x)} = \cot(3x)$$

Ans: We have,

$$\text{L.H.S.} = \frac{(\cos(4x) + \cos(2x)) + \cos(3x)}{(\sin(4x) + \sin(2x)) + \sin(3x)}$$

$$= \frac{2 \cdot \cos\left(\frac{4x+2x}{2}\right) \cdot \cos\left(\frac{4x-2x}{2}\right) + \cos(3x)}{2 \cdot \sin\left(\frac{4x+2x}{2}\right) \cdot \cos\left(\frac{4x-2x}{2}\right) + \sin(3x)}$$

$$= \frac{2 \cdot \cos\left(\frac{4x+2x}{2}\right) \cdot \cos\left(\frac{4x-2x}{2}\right) + \sin(3x)}{2 \cdot \sin\left(\frac{4x+2x}{2}\right) \cdot \cos\left(\frac{4x-2x}{2}\right) + \sin(3x)}$$

$$\left\{ \begin{array}{l} \therefore \cos C + \cos D = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \\ \sin C + \sin D = 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \end{array} \right\}$$

$$= \frac{2 \cdot \cos(3x) \cdot \cos(x) + \cos(3x)}{2 \cdot \sin(3x) \cdot \cos(x) + \sin(3x)}$$

$$= \frac{\cos(3x) \{ 2 \cdot \cos(x) + 1 \}}{\sin(3x) \{ 2 \cdot \cos(x) + 1 \}}$$

$$= \frac{\cos(3x)}{\sin(3x)} = \cot(3x) = R.H.S.$$

Q. Prove that :-

$$\sin(x) + \sin(3x) + \sin(5x) + \sin(7x) = 4 \sin(4x) \cdot \cos(2x) \cdot \cos(x)$$

Ans: L.H.S.

$$= (\sin(7x) + \sin(x)) + (\sin(5x) + \sin(3x))$$

$$= 2 \sin\left(\frac{7x+x}{2}\right) \cdot \cos\left(\frac{7x-x}{2}\right) +$$

$$2 \sin\left(\frac{5x+3x}{2}\right) \cdot \cos\left(\frac{5x-3x}{2}\right)$$

$$= 2 \sin(4x) \cdot \cos(3x) + 2 \sin(4x) \cdot \cos(x)$$

$$= 2 \sin(4x) \times \{ \cos(3x) + \cos(x) \}$$

$$= 2 \sin(4x) \times 2 \cos\left(\frac{3x+x}{2}\right) \cdot \cos\left(\frac{3x-x}{2}\right)$$

$$= 2 \sin(4x) \times 2 \cos(2x) \cdot \cos(x)$$

$$= 4 \sin(4x) \cdot \cos(2x) \cdot \cos(x)$$

$$= R.H.S.$$

Q. Prove that: -

$$\cos(2x) \cdot \cos\left(\frac{x}{2}\right) - \cos(3x) \cdot \cos\left(\frac{9x}{2}\right) = -\sin(5x) \cdot \sin\left(\frac{5x}{2}\right)$$

Ans: $\frac{L.H.S.}{= \frac{1}{2} \left[2 \cdot \cos(2x) \cdot \cos\left(\frac{x}{2}\right) - 2 \cdot \cos(3x) \cdot \cos\left(\frac{9x}{2}\right) \right]}$

$$= \frac{1}{2} \left[\cos\left(2x + \frac{x}{2}\right) + \cos\left(2x - \frac{x}{2}\right) \right] -$$

$$\frac{1}{2} \left[\cos\left(\frac{9x}{2} + 3x\right) + \cos\left(\frac{9x}{2} - 3x\right) \right]$$

$$= \frac{1}{2} \left[\cos\left(\frac{5x}{2}\right) + \cos\left(\frac{3x}{2}\right) \right] -$$

$$\frac{1}{2} \left[\cos\left(\frac{15x}{2}\right) + \cos\left(\frac{3x}{2}\right) \right]$$

$$= \frac{1}{2} \left[\cos\left(\frac{5x}{2}\right) - \cos\left(\frac{15x}{2}\right) \right]$$

$$= \frac{1}{2} \left[-2 \cdot \sin\left(\frac{\frac{5x}{2} + \frac{15x}{2}}{2}\right) \cdot \sin\left(\frac{\frac{15x}{2} - \frac{5x}{2}}{2}\right) \right]$$

$$= -\sin(5x) \cdot \sin\left(\frac{5x}{2}\right)$$

$$= R.H.S. \quad \underline{\underline{=}}$$

Q. Prove that : —

$$\frac{\sin(8x) \cdot \cos(x) - \sin(6x) \cdot \cos(3x)}{\cos(2x) \cdot \cos(x) - \sin(4x) \cdot \sin(3x)} = \tan(2x)$$

Ans: L.H.S.

Multiplying n^o & d^o by 2, we get

$$= \frac{2 \cdot \sin(8x) \cdot \cos(x) - 2 \sin(6x) \cdot \cos(3x)}{2 \cdot \cos(2x) \cdot \cos(x) - 2 \sin(4x) \cdot \sin(3x)}$$

$$\left\{ \begin{array}{l} \because 2 \sin A \cdot \cos B = \sin(A+B) + \sin(A-B) \\ 2 \cos A \cdot \cos B = \cos(A+B) + \cos(A-B) \\ 2 \sin A \cdot \sin B = \cos(A-B) - \cos(A+B) \end{array} \right.$$

$$= \frac{\{\sin(8x+x) + \sin(8x-x)\} - \{\sin(6x+3x) + \sin(6x-3x)\}}{\{\cos(2x+x) + \cos(2x-x)\} - \{\cos(4x-3x) - \cos(4x+3x)\}}$$

$$= \frac{\{\sin(9x) + \sin(7x)\} - \{\sin(9x) + \sin(3x)\}}{\{\cos(3x) + \cos(x)\} - \{\cos(x) - \cos(7x)\}}$$

$$= \frac{\sin(9x) + \sin(7x) - \sin(9x) - \sin(3x)}{\cos(3x) + \cos(x) - \cos(x) + \cos(7x)}$$

$$= \frac{\sin(7x) - \sin(3x)}{\cos(3x) + \cos(7x)}$$

$$= \frac{\sin(7x) - \sin(3x)}{\cos(3x) + \cos(7x)}$$

$$= \frac{\sin(7x) - \sin(3x)}{\cos(3x) + \cos(7x)}$$

$$\begin{cases} \therefore \sin C - \sin D = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \\ \cos C + \cos D = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \end{cases}$$

$$\begin{aligned} &= \frac{2 \cdot \cos\left(\frac{7\pi+3\pi}{2}\right) \cdot \sin\left(\frac{7\pi-3\pi}{2}\right)}{2 \cdot \cos\left(\frac{7\pi+3\pi}{2}\right) \cdot \cos\left(\frac{7\pi-3\pi}{2}\right)} \\ &= \frac{\cos(5\pi) \cdot \sin(2\pi)}{\cos(5\pi) \cdot \cos(2\pi)} = \tan(2\pi) \\ &= \text{R.H.S.} \end{aligned}$$

Q. Prove that :-

$$\frac{\sin(x) - \sin(y)}{\cos(x) + \cos(y)} = \tan\left(\frac{x-y}{2}\right)$$

Ans: L.H.S.

$$\begin{aligned} &= \frac{\sin(x) - \sin(y)}{\cos(x) + \cos(y)} \\ &= \frac{2 \cdot \cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)}{2 \cdot \cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)} \\ &= \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.} \end{aligned}$$

Q. Prove that :—

$$(\cos(x) + \cos(y))^2 + (\sin(x) + \sin(y))^2 = 4 \cdot \cos^2\left(\frac{x-y}{2}\right)$$

Ans: L.H.S.

$$= (\cos(x) + \cos(y))^2 + (\sin(x) + \sin(y))^2$$

$$\left\{ \begin{aligned} \because \cos C + \cos D &= 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \\ \sin C + \sin D &= 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \end{aligned} \right\}$$

$$= \left\{ 2 \cdot \cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right) \right\}^2 +$$

$$\left\{ 2 \cdot \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right) \right\}^2$$

$$= 4 \cdot \cos^2\left(\frac{x+y}{2}\right) \cdot \cos^2\left(\frac{x-y}{2}\right) +$$

$$4 \cdot \sin^2\left(\frac{x+y}{2}\right) \cdot \cos^2\left(\frac{x-y}{2}\right)$$

$$= 4 \cdot \cos^2\left(\frac{x-y}{2}\right) \left\{ \cos^2\left(\frac{x+y}{2}\right) + \sin^2\left(\frac{x+y}{2}\right) \right\}$$

$$= 4 \cdot \cos^2\left(\frac{x-y}{2}\right) \quad \left[\because \sin^2\theta + \cos^2\theta = 1 \right]$$

$$= \text{R.H.S.}$$

Q Prove that :-

$$\cos(\alpha) + \cos(\beta) + \cos(\gamma) + \cos(\alpha + \beta + \gamma)$$

$$= 4 \cdot \cos\left(\frac{\alpha + \beta}{2}\right) \cdot \cos\left(\frac{\beta + \gamma}{2}\right) \cdot \cos\left(\frac{\gamma + \alpha}{2}\right)$$

Ans: L.H.S.

$$= (\cos(\alpha) + \cos(\beta)) + (\cos(\alpha + \beta + \gamma) + \cos(\gamma))$$

$$= 2 \cdot \cos\left(\frac{\alpha + \beta}{2}\right) \cdot \cos\left(\frac{\alpha - \beta}{2}\right) +$$

$$2 \cdot \cos\left(\frac{\alpha + \beta + 2\gamma}{2}\right) \cdot \cos\left(\frac{\alpha + \beta}{2}\right)$$

$$= 2 \cdot \cos\left(\frac{\alpha + \beta}{2}\right) \cdot \left\{ \cos\left(\frac{\alpha - \beta}{2}\right) + \cos\left(\frac{\alpha + \beta + 2\gamma}{2}\right) \right\}$$

$$= 2 \cdot \cos\left(\frac{\alpha + \beta}{2}\right) \cdot \left\{ 2 \cdot \cos\left(\frac{\gamma + \alpha}{2}\right) \cdot \cos\left(\frac{\beta + \gamma}{2}\right) \right\}$$

$$= 4 \cdot \cos\left(\frac{\alpha + \beta}{2}\right) \cdot \cos\left(\frac{\beta + \gamma}{2}\right) \cdot \cos\left(\frac{\beta + \gamma}{2}\right)$$

$$= R.H.S.$$

(P.T.O.)

Q. Prove that :-

$$\sin^2(6x) - \sin^2(4x) = \sin(10x) \cdot \sin(2x)$$

Ans:

L.H.S.

$$= \sin^2(6x) - \sin^2(4x)$$

$$= \sin(6x+4x) \sin(6x-4x)$$

$$= \sin(10x) \cdot \sin(2x)$$

Q. Prove that :-

$$\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{1}{16}$$

Ans:

L.H.S.

$$= \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ$$

$$= \frac{1}{2} \cos 60^\circ \cdot \cos^{40^\circ} \cdot \cos 80^\circ \cdot \cos 20^\circ$$

$$= \frac{1}{2} \times \frac{1}{2} \times \cos 40^\circ \cdot \{ \cos(80^\circ+20^\circ) + \cos(80^\circ-20^\circ) \}$$

$$= \frac{1}{4} \times \cos(40^\circ) \cdot (\cos(100^\circ) + \cos(60^\circ))$$

$$= \frac{1}{4} \times \cos(40^\circ) \left(\cos(100^\circ) + \frac{1}{2} \right)$$

$$= \frac{1}{4} \times \cos(40^\circ) \cdot \cos(100^\circ) + \frac{1}{8} \cdot \cos 40^\circ$$

$$= \frac{1}{8} \times (2 \cos(40^\circ) \cdot \cos(100^\circ)) + \frac{1}{8} \cdot \cos 40^\circ$$

$$= \frac{1}{8} [2 \cos(40^\circ+100^\circ) + \cos(100^\circ-40^\circ)] + \frac{1}{8} \cos 40^\circ$$

$$\begin{aligned}
 &= \frac{1}{8} [\cos(140^\circ) + \cos(60^\circ)] + \frac{1}{8} \cdot \cos(40^\circ) \\
 &= \frac{1}{8} \cdot \cos(140^\circ) + \frac{1}{8} \cdot \cos(60^\circ) + \frac{1}{8} \cdot \cos(40^\circ) \\
 &= \frac{1}{8} \cdot \cos(180^\circ - 40^\circ) + \frac{1}{16} + \frac{1}{8} \cdot \cos(40^\circ) \\
 &= -\frac{1}{8} \cos(40^\circ) + \frac{1}{16} + \frac{1}{8} \cos(40^\circ) \\
 &= \frac{1}{16} = R \cdot H \cdot S =
 \end{aligned}$$

Q. Prove that :-

$$\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \frac{\sqrt{3}}{16}$$

Ans: L.H.S.

$$\begin{aligned}
 &= \frac{1}{2} \cdot \sin 60^\circ \sin 10^\circ (2 \cdot \sin 70^\circ \sin 50^\circ) \\
 &= \frac{1}{2} \times \frac{\sqrt{3}}{2} \times \sin 10^\circ \{ \cos(70^\circ - 50^\circ) - \cos(120^\circ) \} \\
 &= \frac{\sqrt{3}}{4} \times \sin(10^\circ) \{ \cos(20^\circ) - \cos(120^\circ) \} \\
 &= \frac{\sqrt{3}}{4} \times \sin(10^\circ) \left(\cos(20^\circ) + \frac{1}{2} \right) \left\{ \because \cos(120^\circ) = -\frac{1}{2} \right\} \\
 &= \frac{\sqrt{3}}{4} \times \sin(10^\circ) \cdot \cos(20^\circ) + \frac{\sqrt{3}}{8} \sin(10^\circ)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\sqrt{3}}{8} \cdot (2 \cdot \cos 20^\circ \cdot \sin 10^\circ) + \frac{\sqrt{3}}{8} \cdot \sin(10^\circ) \\
 &= \frac{\sqrt{3}}{8} [\sin(20^\circ + 10^\circ) - \sin(20^\circ - 10^\circ)] + \frac{\sqrt{3}}{8} \cdot \sin(10^\circ) \\
 &= \frac{\sqrt{3}}{8} \sin 30^\circ - \frac{\sqrt{3}}{8} \cdot \cancel{\sin(10^\circ)} + \frac{\sqrt{3}}{8} \cdot \cancel{\sin(10^\circ)} \\
 &= \left(\frac{\sqrt{3}}{8} \times \frac{1}{2} \right) = \frac{\sqrt{3}}{16} = \text{R.H.S.}
 \end{aligned}$$

Q. Prove that :-

$$\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan(x)}{1 - \tan(x)}$$

Ans: L.H.S.

$$= \tan\left(\frac{\pi}{4} + x\right)$$

$$= \frac{\tan\left(\frac{\pi}{4}\right) + \tan(x)}{1 - \tan\left(\frac{\pi}{4}\right) \cdot \tan(x)} = \frac{1 + \tan(x)}{1 - \tan(x)} = \text{R.H.S.}$$

Q. Prove that :-

$$\tan\left(\frac{\pi}{4} - x\right) = \frac{1 - \tan(x)}{1 + \tan(x)}$$

Ans: L.H.S.

$$= \tan\left(\frac{\pi}{4} - x\right) = \frac{\tan\left(\frac{\pi}{4}\right) - \tan(x)}{1 + \tan\left(\frac{\pi}{4}\right) \cdot \tan(x)}$$

$$= \frac{1 - \tan(x)}{1 + \tan(x)} = \text{R.H.S.}$$

Q Prove that :-

$$\frac{\sin(x) + \sin(3x)}{\cos(x) - \cos(3x)} = \cot(x)$$

Ans:

L.H.S.

$$= \frac{2 \sin\left(\frac{3x+x}{2}\right) \cdot \cos\left(\frac{3x-x}{2}\right)}{2 \cdot \sin\left(\frac{3x+x}{2}\right) \cdot \sin\left(\frac{3x-x}{2}\right)}$$

$$= \frac{\cos\left(\frac{2x}{2}\right)}{\sin\left(\frac{2x}{2}\right)} = \frac{\cos(x)}{\sin(x)} = \cot(x)$$

$$= \text{R.H.S.}$$

Q. Prove that :-

$$\frac{\sin(7x) - \sin(5x)}{\cos(7x) + \cos(5x)} = \tan(x)$$

Ans:

L.H.S.

$$= \frac{2 \cos\left(\frac{7x+5x}{2}\right) \cdot \sin\left(\frac{7x-5x}{2}\right)}{2 \cos\left(\frac{7x+5x}{2}\right) \cdot \cos\left(\frac{7x-5x}{2}\right)}$$

$$= \frac{2 \cos(6x) \cdot \sin(x)}{2 \cos(6x) \cdot \cos(x)} = \tan(x)$$

$$= \text{R.H.S.}$$

Q. Prove that :-

$$\frac{\sin(x) + \sin(3x) + \sin(5x)}{\cos(x) + \cos(3x) + \cos(5x)} = \tan(3x)$$

Ans:

L.H.S.

$$= \frac{(\sin(5x) + \sin(x)) + \sin(3x)}{(\cos(5x) + \cos(x)) + \cos(3x)}$$

$$= \frac{2 \cdot \sin\left(\frac{5x+x}{2}\right) \cdot \cos\left(\frac{5x-x}{2}\right) + \sin(3x)}{2 \cdot \cos\left(\frac{5x+x}{2}\right) \cdot \cos\left(\frac{5x-x}{2}\right) + \cos(3x)}$$

$$= \frac{2 \cdot \sin(3x) \cdot \cos(2x) + \sin(3x)}{2 \cdot \cos(3x) \cdot \cos(2x) + \cos(3x)}$$

$$= \frac{\sin(3x) [2 \cdot \cos(2x) + 1]}{\cos(3x) [2 \cdot \cos(2x) + 1]}$$

$$= \tan(3x) = R.H.S.$$

Q. Prove that :-

$$\frac{(\sin(7x) + \sin(5x)) + (\sin(9x) + \sin(3x))}{(\cos(7x) + \cos(5x)) + (\cos(9x) + \cos(3x))} = \tan(6x)$$

Ans: L.H.S.

$$\begin{aligned} & 2 \cdot \sin\left(\frac{7x+5x}{2}\right) \cdot \cos\left(\frac{7x-5x}{2}\right) + 2 \cdot \sin\left(\frac{9x+3x}{2}\right) \cdot \cos\left(\frac{9x-3x}{2}\right) \\ &= 2 \cdot \cos\left(\frac{7x+5x}{2}\right) \cdot \cos\left(\frac{7x-5x}{2}\right) + 2 \cdot \cos\left(\frac{9x+3x}{2}\right) \cdot \cos\left(\frac{9x-3x}{2}\right) \\ &= 2 \cdot \sin(6x) \cdot \cos(x) + 2 \cdot \sin(6x) \cdot \cos(3x) \\ &= 2 \cdot \cos(6x) \cdot \cos(x) + 2 \cdot \cos(6x) \cdot \cos(3x) \\ &= 2 \cdot \sin(6x) (\cos(x) + \cos(3x)) \\ &= 2 \cdot \cos(6x) (\cos(x) + \cos(3x)) \\ &= \frac{\sin(6x)}{\cos(6x)} = \tan(6x) = R.H.S. \end{aligned}$$

Q. Prove that: —

$$\cot(4x) \cdot (\sin(5x) + \sin(3x)) = \cot(x) (\sin(5x) - \sin(3x))$$

Ans:

L.H.S.

$$\begin{aligned} &= \cot(4x) \cdot (\sin(5x) + \sin(3x)) \\ &= \cot(4x) \left(2 \cdot \sin\left(\frac{5x+3x}{2}\right) \cdot \cos\left(\frac{5x-3x}{2}\right) \right) \\ &= \cot(4x) (2 \cdot \sin(4x) \cdot \cos(x)) \\ &= \frac{\cos(4x)}{\sin(4x)} \times 2 \times \cancel{\sin(4x)} \times \cos(x) \\ &= 2 \cdot \cos(4x) \cdot \cos(x) \\ &= R.H.S. \end{aligned}$$

Q. Prove that :-

$$(\sin 3x + \sin x) \cdot \sin x + (\cos 3x - \cos x) \cdot \cos x = 0.$$

Ans:

$$\begin{cases} \because \sin C + \sin D = 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \\ \cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{D-C}{2}\right) \end{cases}$$

∴ H.O.So

$$= \left(2 \cdot \sin\left(\frac{3x+x}{2}\right) \cdot \cos\left(\frac{3x-x}{2}\right) \right) \cdot \sin x$$

$$+ \left(2 \sin\left(\frac{3x+x}{2}\right) \cdot \sin\left(\frac{3x-x}{2}\right) \right) \cdot \cos x$$

$$= (2 \cdot \sin(2x) \cdot \cos(x)) \cdot \sin x - (2 \cdot \sin(2x) \cdot \sin(x)) \cdot \cos x$$

$$= 0$$

Q. Prove that :-

$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2$$

$$= 4 \cdot \sin^2\left(\frac{x-y}{2}\right)$$

(P.T.O.)

Ans: L.H.S.

$$= \left\{ \begin{aligned} \because \cos C - \cos D &= 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{D-C}{2}\right) \\ \sin C - \sin D &= 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \end{aligned} \right\}$$

$$= \left(-2 \cdot \sin\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right) \right)^2 +$$

$$\left(2 \cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right) \right)^2$$

$$= 4 \cdot \sin^2\left(\frac{x-y}{2}\right) \left[\cancel{\sin^2\left(\frac{x+y}{2}\right)} + \cos^2\left(\frac{x+y}{2}\right) \right]$$

$$= 4 \cdot \sin^2\left(\frac{x-y}{2}\right) = R.H.S$$

proven

Q. Prove that :-

$$\frac{\sin(2x) - \sin(2y)}{\cos(2y) - \cos(2x)} = \cot(x+y)$$

Ans:

L.H.S.

$$2 \cdot \cos\left(\frac{2x+2y}{2}\right) \cdot \sin\left(\frac{2x-2y}{2}\right)$$

Q

$$= \frac{2 \cdot \sin\left(\frac{2y+2x}{2}\right) \cdot \sin\left(\frac{2x-2y}{2}\right)}{2 \cdot \sin\left(\frac{2y+2x}{2}\right) \cdot \sin\left(\frac{2x-2y}{2}\right)}$$

$$\begin{aligned}
 &= \frac{2 \cos(x+y) \cdot \sin(x-y)}{2 \sin(x+y) \cdot \sin(x-y)} \\
 &= \cot(x+y) = R.H.S.
 \end{aligned}$$

Q. Prove that :-

$$\frac{\cos(x) + \cos(y)}{\cos(y) - \cos(x)} = \cot\left(\frac{x+y}{2}\right) \cdot \cot\left(\frac{x-y}{2}\right)$$

Ans: L.H.S.

$$\begin{aligned}
 &= \frac{2 \cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)}{2 \sin\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)} \\
 &= \cot\left(\frac{x+y}{2}\right) \cdot \cot\left(\frac{x-y}{2}\right) \\
 &= R.H.S.
 \end{aligned}$$

Q. Prove that :-

$$\frac{\sin(x) + \sin(y)}{\sin(x) - \sin(y)} = \tan\left(\frac{x+y}{2}\right) \cdot \cot\left(\frac{x-y}{2}\right)$$

(P.T.O.)

Ans: L.H.S.

$$\frac{\sin(x) + \sin(y)}{\sin(x) - \sin(y)}$$

$$\left\{ \begin{aligned} \because \sin C + \sin D &= 2 \cdot \sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \\ \sin C - \sin D &= 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right) \end{aligned} \right\}$$

$$= \frac{2 \cdot \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)}{2 \cdot \cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x+y}{2}\right) \cdot \cot\left(\frac{x-y}{2}\right)$$

$$\textcircled{1}. \sin(3x) + \sin(2x) - \sin(x) = 4 \cdot \sin(x) \cdot \cos\left(\frac{x}{2}\right) \cdot \cos\left(\frac{3x}{2}\right)$$

Ans: L.H.S.

$$= \sin(3x) - \sin(x) + \sin(2x)$$

$$= 2 \cdot \cos\left(\frac{3x+x}{2}\right) \cdot \sin\left(\frac{3x-x}{2}\right) + \sin(2x)$$

$$= 2 \cdot \cos(2x) \cdot \sin(x) + \sin(2x)$$

$$= 2 \cdot \cos(2x) \cdot \sin(x) + 2 \sin(x) \cdot \cos(x)$$

$$= 2 \cdot \sin(x) [\cos(2x) + \cos(x)]$$

$$= 2 \cdot \sin(x) \left[2 \cos\left(\frac{2x+x}{2}\right) \cdot \cos\left(\frac{2x-x}{2}\right) \right]$$

$$= 4 \cdot \sin(x) \cdot \cos\left(\frac{3x}{2}\right) \cdot \cos\left(\frac{x}{2}\right) = \text{R.H.S.}$$

Q. Prove that :-

$$\frac{\cos(4x) \cdot \sin(3x) - \cos(2x) \cdot \sin(x)}{\sin(4x) \cdot \sin(x) + \cos(6x) \cdot \cos(x)} = \tan(2x)$$

Ans L.H.S.

$$= \frac{2 \cdot \cos(4x) \cdot \sin(3x) - 2 \cdot \cos(2x) \cdot \sin(x)}{2 \cdot \sin(4x) \cdot \sin(x) + 2 \cos(6x) \cdot \cos(x)}$$

$$= \frac{\sin(4x+3x) - \sin(4x-3x) - \{\sin(2x+x) - \sin(2x-x)\}}{\cos(4x-x) - \cos(4x+x) + \cos(6x+x) + \cos(6x-x)}$$

$$= \frac{\sin(7x) - \sin(x) - \sin(3x) + \sin(x)}{\cos(3x) - \cos(5x) + \cos(7x) + \cos(5x)}$$

$$= \frac{\sin(7x) - \sin(3x)}{\cos(3x) + \cos(7x)}$$

$$= \frac{2 \cos\left(\frac{7x+3x}{2}\right) \cdot \sin\left(\frac{7x-3x}{2}\right)}{2 \cos\left(\frac{3x+7x}{2}\right) \cdot \cos\left(\frac{3x-7x}{2}\right)}$$

$$= \frac{\sin(2x)}{\cos(2x)}$$

$$= \tan(2x) = \text{R.H.S.}$$

Q. Prove that :-

$$\sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ = \frac{1}{16}$$

Ans: L.H.S

$$= \sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ$$

$$= \frac{1}{2} (2 \cdot \sin 70^\circ \cdot \sin 10^\circ) \cdot \sin 50^\circ \times \frac{1}{2}$$

$$= \frac{1}{4} [\cos(70^\circ - 10^\circ) - \cos(70^\circ + 10^\circ)] \sin 50^\circ$$

$$= \frac{1}{4} [\cos 60^\circ \cdot \sin 50^\circ - \cos 80^\circ \cdot \sin 50^\circ]$$

$$= \frac{1}{4} \left[\frac{1}{2} \cdot \sin 50^\circ - \cos 80^\circ \cdot \sin 50^\circ \right]$$

$$= \frac{1}{8} [\sin 50^\circ - 2 \cdot \cos 80^\circ \cdot \sin 50^\circ]$$

$$\# \left\{ \because 2 \cdot \cos A \cdot \sin B = \sin(A+B) - \sin(A-B) \right\}$$

$$= \frac{1}{8} [\sin 50^\circ - \sin(80^\circ + 50^\circ) + \sin(80^\circ - 50^\circ)]$$

$$= \frac{1}{8} [\sin 50^\circ - \sin(130^\circ) + \sin(30^\circ)]$$

$$= \frac{1}{8} \left[\sin 50^\circ - \sin(130^\circ) + \frac{1}{2} \right]$$

$$= \frac{1}{8} [\cancel{\sin 50^\circ} - \cancel{\sin(180^\circ - 50^\circ)} + \frac{1}{2}] = \frac{1}{16}$$

= R.H.S

Q. Prove that :-

$$\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = \frac{3}{16}$$

Ans:

$$\text{L.H.S.} = \sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ$$

$$= \frac{1}{2} (2 \cdot \sin 80^\circ \cdot \sin 20^\circ) \cdot \sin 40^\circ \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} (2 \cdot \sin 80^\circ \cdot \sin 20^\circ) \cdot \sin 40^\circ$$

$$\left\{ \because 2 \sin A \sin B = \cos(A-B) - \cos(A+B) \right\}$$

$$= \frac{\sqrt{3}}{4} [\cos(80^\circ - 20^\circ) - \cos(80^\circ + 20^\circ)] \cdot \sin(40^\circ)$$

$$= \frac{\sqrt{3}}{4} [\cos(60^\circ) - \cos(100^\circ)] \cdot \sin(40^\circ)$$

$$= \frac{\sqrt{3}}{4} [\cos(60^\circ) \cdot \sin(40^\circ) - \cos(100^\circ) \cdot \sin(40^\circ)]$$

$$= \frac{\sqrt{3}}{4} \left[\frac{1}{2} \cdot \sin(40^\circ) - \cos(100^\circ) \cdot \sin(40^\circ) \right]$$

$$= \frac{\sqrt{3}}{8} [\sin(40^\circ) - 2 \cos(100^\circ) \cdot \sin(40^\circ)]$$

$$= \frac{\sqrt{3}}{8} [\sin(40^\circ) - \sin(140^\circ) + \sin(60^\circ)]$$

$$= \frac{\sqrt{3}}{8} \left[\cancel{\sin 40^\circ} - \cancel{\sin(180^\circ - 40^\circ)} + \sin 60^\circ \right]$$

$$= \frac{\sqrt{3}}{8} \left[\frac{\sqrt{3}}{2} \right] = \frac{3}{16} = \text{R.H.S.}$$

Q. Prove that :-

$$\cos(10^\circ) \cdot \cos(30^\circ) \cdot \cos(50^\circ) \cdot \cos(70^\circ) = \frac{3}{16}$$

Ans: L.H.S.

$$= \cos(10^\circ) \cdot \cos(30^\circ) \cdot \cos(50^\circ) \cdot \cos(70^\circ)$$

$$= \frac{1}{2} (2 \cdot \cos(70^\circ) \cdot \cos(10^\circ)) \cdot \cos(50^\circ) \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} [2 \cdot \cos(70^\circ) \cdot \cos(10^\circ)] \cdot \cos(50^\circ)$$

$$= \frac{\sqrt{3}}{4} [\cos(70^\circ + 10^\circ) + \cos(70^\circ - 10^\circ)] \cdot \cos(50^\circ)$$

$$= \frac{\sqrt{3}}{4} [\cos(80^\circ) + \cos(60^\circ)] \cdot \cos(50^\circ)$$

$$= \frac{\sqrt{3}}{4} [\cos(80^\circ) \cdot \cos(50^\circ) + \cos(60^\circ) \cdot \cos(50^\circ)]$$

$$= \frac{\sqrt{3}}{4} \left[\cos(80^\circ) \cdot \cos(50^\circ) + \frac{1}{2} \cdot \cos(50^\circ) \right]$$

$$= \frac{\sqrt{3}}{8} [2 \cdot \cos(80^\circ) \cdot \cos(50^\circ) + \cos(50^\circ)]$$

$$= \frac{\sqrt{3}}{8} \left[2 \cdot \cos(80^\circ + 50^\circ) \cdot \cos(80^\circ - 50^\circ) + \cos 50^\circ \right]$$

$$= \frac{\sqrt{3}}{8} \left[2 \cdot \cos(130^\circ) \cdot \cos(30^\circ) + \cos(50^\circ) \right]$$

$$= \frac{\sqrt{3}}{8} \left[2 \cdot \cos(130^\circ) \times \frac{\sqrt{3}}{2} + \cos 50^\circ \right]$$

$$= \frac{\sqrt{3}}{8} \left[\cos(180^\circ - 50^\circ) + \cos(30^\circ) + \cos(50^\circ) \right]$$

$$= \frac{\sqrt{3}}{8} \left[-\cos(50^\circ) + \cos(30^\circ) + \cos(50^\circ) \right]$$

$$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16} = \text{R.H.S.}$$

Q. If $\cos(x) + \cos(y) = \frac{1}{3}$ and

$\sin(x) + \sin(y) = \frac{1}{4}$, prove that

$$\tan\left(\frac{x+y}{2}\right) = \frac{3}{4}$$

Ans:

$$\Rightarrow \frac{\sin(x) + \sin(y)}{\cos(x) + \cos(y)} = \frac{\frac{1}{4}}{\frac{1}{3}}$$

$$\Rightarrow \frac{\sin(x) + \sin(y)}{\cos(x) + \cos(y)} = \frac{3}{4}$$

$$\Rightarrow \frac{2 \sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)}{2 \cdot \cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)} = \frac{3}{4}$$

$$\therefore \tan\left(\frac{x+y}{2}\right) = \frac{3}{4} \quad \text{proved}$$

Q. If $\sin(x) = -\frac{1}{2}$ and $\pi < x < \frac{3\pi}{2}$, find the values of
(i) $\sin(2x)$ (ii) $\cos(2x)$ (iii) $\tan(2x)$

Ans: $\because x$ lies in Quadrant IIIrd, we have
 $\sin(x) < 0$, $\cos(x) < 0$ & $\tan(x) > 0$.

Now, $\sin(x) = -\frac{1}{2}$ (given)

$$\therefore \cos(x) = -\sqrt{1 - \sin^2(x)}$$

$$= -\sqrt{\left(1 - \frac{1}{4}\right)} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$$

$$\& \tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{-\frac{1}{2} \times \frac{2}{\sqrt{3}}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

(i) $\sin(2x)$

$$= 2 \sin(x) \cos(x)$$

$$= 2 \times \left(-\frac{1}{2}\right) \times \left(-\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{2}$$

$$(ii.) \cos(2x) = (2 \cdot \cos^2(x) - 1)$$

$$= \left[2 \times \left(-\frac{\sqrt{3}}{2} \right)^2 - 1 \right]$$

$$= \left[2 \times \frac{3}{4} - 1 \right]$$

$$= \frac{3}{2} - 1 = \frac{3-2}{2} = \frac{1}{2}$$

$$(iii.) \tan(2x)$$

$$= \frac{\sin(2x)}{\cos(2x)} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$$

Q. If $\sin(x) = \frac{1}{3}$, find the value of $\sin(3x)$.

Ans: $\sin(3x) = 3 \cdot \sin(x) - 4 \cdot \sin^3(x)$

$$= 3 \cdot \left(\frac{1}{3} \right) - 4 \cdot \left(\frac{1}{3} \right)^3$$

$$= 1 - \frac{4}{27} = \frac{27-4}{27} = \frac{23}{27}$$

Q. If $\cos(x) = \frac{1}{2}$, find the value of $\cos(3x)$.

Ans: $\cos(3x) = 4 \cdot \cos^3(x) - 3 \cdot \cos(x)$

$$= \left\{ 4 \times \left(\frac{1}{2} \right)^3 - 3 \times \frac{1}{2} \right\} = \left(\frac{1}{2} - \frac{3}{2} \right)$$

$$= -1$$

Q. If $\cos(x) = \frac{4}{5}$ and x is acute, find the value of $\tan(2x)$.

Ans: $\cos(x) = \frac{4}{5} \Rightarrow \sec(x) = \frac{5}{4}$

$$\Rightarrow \tan(x) = \sqrt{\sec^2(x) - 1} = \sqrt{\frac{25}{16} - 1}$$

$$= \sqrt{\frac{25-16}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$$\therefore \tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)} = \frac{2 \times \frac{3}{4}}{1 - \frac{9}{16}}$$

$$\Rightarrow \frac{\frac{6}{4}}{\frac{7}{16}} = \frac{\cancel{6}^3 \times \cancel{16}^8}{\cancel{4}_2 \times 7} = \frac{24}{7}$$

Q. If $\tan(x) = \frac{1}{7}$ and $\tan(y) = \frac{1}{3}$, show that

$$\cos(2x) = \sin(4y).$$

Ans: $\cos(2x) = \frac{1 - \tan^2(x)}{1 + \tan^2(x)} = \frac{\left(1 - \frac{1}{49}\right)}{\left(1 + \frac{1}{49}\right)}$

$$\Rightarrow \frac{49-1}{49+1} = \frac{\cancel{48}^{24}}{\cancel{50}^{25}} = \frac{24}{25}$$

(P.T.O.)

$$\sin(4y) = 2 \cdot \sin(2y) \cdot \cos(2y)$$

$$= 2 \times \frac{2 \tan(y)}{(1 + \tan^2(y))} \times \frac{1 - \tan^2(y)}{1 + \tan^2(y)}$$

$$= 2 \times \frac{(2 \times \frac{1}{3})}{1 + (\frac{1}{3})^2} \times \frac{1 - (\frac{1}{3})^2}{1 + (\frac{1}{3})^2}$$

$$= \left(\frac{4}{3} \times \frac{9}{10} \times \frac{8}{9} \times \frac{9}{10} \right) = \frac{24}{25}$$

$$\therefore \cos(2x) = \sin(4y)$$

Q. Prove that :-

$$\sin\left(\frac{\pi}{6}\right) \cdot \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{4}$$

Ans: L.H.S

$$= \frac{1}{2} \times \left(2 \cdot \sin\left(\frac{\pi}{6}\right) \cdot \cos\left(\frac{\pi}{6}\right) \right)$$

$$= \frac{1}{2} \times \sin\left(2 \times \frac{\pi}{6}\right)$$

$$= \frac{1}{2} \times \sin\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} = \text{R.H.S.}$$

$\left[\begin{array}{l} \because 2 \sin(x) \cdot \cos(x) \\ = \sin(2x) \end{array} \right]$

Q. Prove that :-

$$\cos^2\left(\frac{\pi}{12}\right) - \sin^2\left(\frac{\pi}{12}\right)$$

Ans: $\left\{ \because \cos^2(x) - \sin^2(x) = \cos(2x) \right\}$

$$\Rightarrow \cos\left(2 \times \frac{\pi}{12}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

Q. Prove that :-

$$\frac{\sin(2x)}{1 - \cos(2x)} = \cot(x)$$

Ans: L.H.S.

$$\Rightarrow \frac{\sin(2x)}{1 - \cos(2x)} = \frac{2 \cdot \sin(x) \cdot \cos(x)}{2 \cdot \sin^2(x)}$$

$$\left\{ \begin{array}{l} \because \sin(2x) = 2 \cdot \sin(x) \cdot \cos(x) \\ 1 - \cos(2x) = 2 \cdot \sin^2(x) \end{array} \right\}$$

$$= \frac{\cos(x)}{\sin(x)} = \cot(x) = \text{R.H.S.} \quad \text{proved}$$

Q. Prove that :-

$$\frac{1 - \cos(2x)}{1 + \cos(2x)} = \tan^2(x)$$

(P.T.O.)

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Page _____

Ans: L.H.S.

$$= \frac{1 - \cos(2x)}{1 + \cos(2x)} \quad \left\{ \begin{array}{l} \because \\ 1 - \cos(2x) = 2 \cdot \sin^2(x) \\ 1 + \cos(2x) = 2 \cdot \cos^2(x) \end{array} \right.$$

$$= \frac{2 \cdot \sin^2(x)}{2 \cdot \cos^2(x)} = \tan^2(x)$$

= R.H.S. proved

Q. Prove that :-

$$\cos(4x) = 1 - 8 \cdot \sin^2(x) \cdot \cos^2(x)$$

Ans: L.H.S.

$$= \cos(4x)$$

$$= \cos 2(2x)$$

let $2x = \theta$

$$= \cos 2\theta$$

$$= \cos^2(\theta) - \sin^2(\theta)$$

$$= (1 - 2 \cdot \sin^2(\theta)) \quad \left\{ \because \cos^2(\theta) = 1 - \sin^2(\theta) \right.$$

putting $\theta = 2x$; we get

$$= \{1 - 2 \cdot \sin^2(2x)\}$$

$$= 1 - 2 (2 \cdot \sin(x) \cdot \cos(x))^2$$

$$= 1 - 2 \times 4 \cdot \sin^2(x) \cdot \cos^2(x)$$

$$= 1 - 8 \cdot \sin^2(x) \cdot \cos^2(x)$$

$$= \text{R.H.S.}$$

proved

Q. Prove that :-

$$\frac{1 + \sin(2x) - \cos(2x)}{1 + \sin(2x) + \cos(2x)} = \tan(x)$$

Ans:

L.H.S.

$$= \frac{1 + \sin(2x) - \cos(2x)}{1 + \sin(2x) + \cos(2x)}$$

$$= \frac{(1 - \cos(2x)) + \sin(2x)}{(1 + \cos(2x)) + \sin(2x)}$$

$$= \frac{2 \cdot \sin^2(x) + 2 \cdot \sin(x) \cdot \cos(x)}{2 \cdot \cos^2(x) + 2 \cdot \sin(x) \cdot \cos(x)}$$

$$= \frac{2 \sin(x) [\cancel{\sin(x)} + \cos(x)]}{2 \cos(x) [\cancel{\cos(x)} + \sin(x)]}$$

$$= \tan(x) = R.H.S.$$

Q. Prove that :-

$$\frac{1 - \sin(2x)}{1 + \sin(2x)} = \tan^2\left(\frac{\pi}{4} - x\right)$$

(P.T.O.)

Ans:

$$\begin{aligned}
 & \text{L.H.S.} \\
 &= \frac{1 - \sin(2x)}{1 + \sin(2x)} \\
 &= \frac{1 - \cos\left(\frac{\pi}{2} - 2x\right)}{1 + \cos\left(\frac{\pi}{2} - 2x\right)} \left\{ \begin{array}{l} \because \sin(2x) = \cos\left(\frac{\pi}{2} - 2x\right) \end{array} \right.
 \end{aligned}$$

$$\left\{ \begin{array}{l} \because 1 - \cos\theta = 2 \cdot \sin^2\left(\frac{\theta}{2}\right) \\ 1 + \cos\theta = 2 \cdot \cos^2\left(\frac{\theta}{2}\right) \end{array} \right.$$

$$= \frac{2 \cdot \sin^2\left(\frac{\pi}{4} - x\right)}{2 \cdot \cos^2\left(\frac{\pi}{4} - x\right)}$$

$$= \frac{\sin^2\left(\frac{\pi}{4} - x\right)}{\cos^2\left(\frac{\pi}{4} - x\right)}$$

$$= \tan^2\left(\frac{\pi}{4} - x\right) = \text{R.H.S.} \quad \text{proved}$$

Q. Prove that :-

$$\frac{\cos(x) + \sin(x)}{\cos(x) - \sin(x)} - \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)} = 2 \cdot \tan(2x)$$

(P.T.O.)

Ans: L.H.S.

$$= \frac{\cos(x) + \sin(x)}{\cos(x) - \sin(x)} - \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)}$$

$$= \frac{(\cos(x) + \sin(x))^2 - (\cos(x) - \sin(x))^2}{\cos^2(x) - \sin^2(x)}$$

$$= \frac{\cancel{\cos^2(x)} + \cancel{\sin^2(x)} + 2 \cdot \sin(x) \cdot \cos(x) - \cancel{\cos^2(x)} - \cancel{\sin^2(x)} + 2 \sin(x) \cdot \cos(x)}{\cos^2(x) - \sin^2(x)}$$

$$= \frac{4 \cdot \sin(x) \cdot \cos(x)}{\cos^2(x) - \sin^2(x)} = \frac{2 \cdot \sin(2x)}{\cos(2x)}$$

$$= 2 \cdot \tan(2x) = \text{R.H.S.}$$

Q. Prove that :-

$$\tan(4\theta) = \frac{4 \cdot \tan(\theta) (1 - \tan^2(\theta))}{1 - 6 \cdot \tan^2(\theta) + \tan^4(\theta)}$$

Ans: L.H.S.

$$= \tan(4\theta)$$

$$= \tan 2(2\theta)$$

$$= \tan 2x, \text{ where } x = 2\theta$$

$$= \frac{2 \tan(x)}{1 - \tan^2(x)} = \frac{2 \cdot \tan(2\theta)}{1 - \tan^2(2\theta)}$$

$$2x \quad \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{4 \tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{1 - \left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right)^2}{\frac{(1 - \tan^2 \theta)^2 - (2 \tan \theta)^2}{(1 - \tan^2 \theta)^2}}$$

$$\begin{aligned}
&= \frac{4 \cdot \tan(\theta) \cdot (1 - \tan^2(\theta))^2}{(1 - \tan^2(\theta)) \{ (1 - \tan^2(\theta))^2 - (2 \tan \theta)^2 \}} \\
&= \frac{4 \cdot \tan(\theta) (1 - \tan^2(\theta))}{(1 - \tan^2(\theta))^2 - 4 \cdot \tan^2(\theta)} \\
&= \frac{4 \cdot \tan(\theta) \cdot (1 - \tan^2(\theta))}{1 + \tan^4 \theta - 2 \tan^2 \theta - 4 \tan^2 \theta} \\
&= \frac{4 \cdot \tan(\theta) \cdot (1 - \tan^2(\theta))}{1 + \tan^4(\theta) - 6 \cdot \tan^2(\theta)} \\
&= R.H.S.
\end{aligned}$$

Q. Prove that :-

$$\frac{\tan(5\theta) + \tan(3\theta)}{\tan(5\theta) - \tan(3\theta)} = 4 \cdot \cos(2\theta) \cdot \cos(4\theta)$$

Ans: L.H.S.

$$= \frac{\tan(5\theta) + \tan(3\theta)}{\tan(5\theta) - \tan(3\theta)}$$

$$= \frac{\left(\frac{\sin(5\theta)}{\cos(5\theta)} \right) + \left(\frac{\sin(3\theta)}{\cos(3\theta)} \right)}{\left(\frac{\sin(5\theta)}{\cos(5\theta)} \right) - \left(\frac{\sin(3\theta)}{\cos(3\theta)} \right)}$$

$$\begin{aligned}
 & \frac{\sin(50) \cdot \cos(30) + \sin(30) \cdot \cos(50)}{\cos(50) \cdot \cos(30)} \\
 = & \frac{\sin(50) \cdot \cos(30) - \sin(30) \cdot \cos(50)}{\cos(50) \cdot \cos(30)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\sin(50) \cdot \cos(30) + \sin(30) \cdot \cos(50)}{\sin(50) \cdot \cos(30) - \sin(30) \cdot \cos(50)}
 \end{aligned}$$

$$\left\{ \begin{aligned} \because \sin A \cdot \cos B + \cos A \cdot \sin B &= \sin(A+B) \\ \sin A \cdot \cos B - \cos A \cdot \sin B &= \sin(A-B) \end{aligned} \right\}$$

$$= \frac{\sin(50+30)}{\sin(50-30)} = \frac{\sin(80)}{\sin(20)}$$

$$= \frac{2 \cdot \sin(40) \cdot \cos(40)}{\sin(20)}$$

$$= \frac{2 \cdot 2 \cdot \sin(20) \cdot \cos(20) \cdot \cos(40)}{\sin(20)}$$

$$= 4 \cdot \cos(20) \cdot \cos(40)$$

$$= R.H.S.$$

(P.T.O.)

Q. Prove that :—

$$\frac{\sec(80) - 1}{\sec(40) - 1} = \frac{\tan(80)}{\tan(20)}$$

Ans:

$$\begin{aligned} & \text{L.H.S.} \\ &= \frac{\sec(80) - 1}{\sec(40) - 1} \end{aligned}$$

$$= \frac{\left(\frac{1}{\cos(80)} - 1\right)}{\left(\frac{1}{\cos(40)} - 1\right)} = \frac{(1 - \cos 80) \cos(40)}{(1 - \cos 40) \cos(80)}$$

$$= \frac{2 \sin^2(40) \cdot \cos(40)}{2 \sin^2(20) \cdot \cos(80)}$$

$$\left[\begin{aligned} \because 1 - \cos 2\theta &= 2 \sin^2 \theta \\ 1 + \cos 2\theta &= 2 \cos^2 \theta \end{aligned} \right]$$

$$= \frac{(2 \cdot \sin(40) \cdot \cos(40)) \cdot \sin(40)}{(2 \cdot \sin^2(20)) \cdot \cos(80)}$$

$$= \frac{\sin(80) \cancel{\sin(20)} \cdot \cos(20)}{\cos(80) \cdot (2 \cdot \sin^2(20))}$$

$$= \cancel{\cot 80} \tan(80) \cdot \cot(20)$$

$$= \frac{\tan(80)}{\tan(20)} = \text{R.H.S.}$$

Q. Show that :-

$$\sqrt{2 + \sqrt{2 + 2 \cdot \cos(4\theta)}} = 2 \cdot \cos(\theta)$$

Ans:- L.H.S.

$$= \sqrt{2 + \sqrt{2 + 2 \cdot \cos(4\theta)}}$$

$$= \sqrt{2 + \sqrt{2(1 + \cos 4\theta)}}$$

$$= \sqrt{2 + \sqrt{2 \cdot 2 \cdot \cos^2(2\theta)}}$$

$$= \sqrt{2 + \sqrt{4 \cdot \cos^2(2\theta)}} \quad \left\{ \begin{array}{l} \because 1 + \cos(2\theta) = 2 \cdot \cos^2(\theta) \end{array} \right.$$

$$= \sqrt{2 + 2 \cdot \cos(2\theta)}$$

$$= \sqrt{2(1 + \cos(2\theta))}$$

$$= \sqrt{2 \cdot 2 \cdot \cos^2(\theta)}$$

$$= \sqrt{4 \cdot \cos^2(\theta)}$$

$$= 2 \cdot \cos(\theta)$$

Q. Show that :-

$$\cos(5x) = 16 \cdot \cos^5(x) - 20 \cdot \cos^3(x) + 5 \cdot \cos(x)$$

Ans:-

L.H.S.

$$= \cos(5x)$$

$$= \cos(3x + 2x)$$

$$= \cos(3x) \cdot \cos(2x) - \sin(3x) \cdot \sin(2x)$$

$$= (4 \cdot \cos^3(x) - 3 \cos(x)) (2 \cdot \cos^2(x) - 1) - (3 \sin(x) - 4 \sin^3(x)) \cdot (2 \sin(x) \cdot \cos(x))$$

$$= \frac{(8 \cdot \cos^5(x) - 10 \cdot \cos^3(x) + 3 \cdot \cos(x))}{(6 \sin^2(x) \cdot \cos(x)) + 8 \sin^4(x) \cdot \cos(x)} -$$

$$= \frac{8 \cos^5(x) - 10 \cos^3(x) + 3 \cos(x)}{6(1 - \cos^2(x)) \cdot \cos(x) + 8(1 - \cos^2(x))^2 \cdot \cos(x)} -$$

$$= \frac{16 \cos^5(x) - 20 \cos^3(x) + 5 \cos(x)}{1} = \text{R.H.S.} \quad \text{proved}$$

Q. Prove that :-

$$\cos(6x) = 32 \cos^6(x) - 48 \cos^4(x) + 18 \cos^2(x) - 1$$

Ans:

L.H.S.

$$= \cos(6x)$$

$$= \cos 2(3x)$$

$$= \cos 2\theta, \text{ where } 3x = \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 2 \cos^2(3x) - 1$$

$$= 2(4 \cos^3(x) - 3 \cos(x))^2 - 1$$

$$= 2(16 \cos^6(x) + 9 \cos^2(x) - 24 \cos^4(x)) - 1$$

$$= 32 \cos^6(x) - 48 \cos^4(x) + 18 \cos^2(x) - 1$$

$$= \text{R.H.S.} \quad \text{proved}$$

Q. Prove that :-

$$\cot(2x) \cdot \cot(x) - \cot(3x) \cdot \cot(2x) - \cot(3x) \cdot \cot(x) = 1$$

Ans: ~~10450~~

$$\Rightarrow \cot(3x) = \cot(2x + x)$$

$$\Rightarrow \cot(3x) = \frac{\cot(2x) \cdot \cot(x) - 1}{\cot(2x) + \cot(x)}$$

$$\Rightarrow \cot(3x) \cdot \cot(2x) + \cot(3x) \cdot \cot(x) = \cot(2x) \cdot \cot(x) - 1$$

$$\Rightarrow \cot(2x) \cdot \cot(x) - \cot(3x) \cdot \cot(2x) - \cot(3x) \cdot \cot(x) = 1$$

proven

★ ★ ★ Find the value of:—

(i) $\sin(18^\circ)$

Let $\theta = 18^\circ$, Then

$$\theta = 18^\circ \Rightarrow 5\theta = 90^\circ$$

$$\Rightarrow 2\theta = (90^\circ - 3\theta)$$

$$\Rightarrow \sin(2\theta) = \sin(90^\circ - 3\theta)$$

$$\Rightarrow 2 \cdot \sin\theta \cdot \cos\theta = \cos(3\theta)$$

$$\Rightarrow 2 \cdot \sin\theta \cdot \cos\theta = 4 \cdot \cos^3(\theta) - 3 \cdot \cos(\theta)$$

$$\Rightarrow 2 \cdot \sin\theta \cdot \cos\theta - 4 \cdot \cos^3(\theta) + 3 \cdot \cos(\theta) = 0$$

$$\Rightarrow \cos(\theta) (2 \cdot \sin(\theta) - 4 \cdot \cos^2(\theta) + 3) = 0$$

{ $\cos\theta = \cos 18^\circ \neq 0$ क्योंकि 0 से 45° में \cos का value 1 से $1/\sqrt{2}$ के बीच में रहता है ; इसलिए इसको discard कर रहे हैं। }

$$\Rightarrow 2 \sin(\theta) - 4 \cos^2(\theta) + 3 = 0$$

$$\Rightarrow 2 \sin(\theta) - 4(1 - \sin^2(\theta)) + 3 = 0$$

$$\Rightarrow 2 \sin(\theta) - 4 + 4 \sin^2(\theta) + 3 = 0$$

$$\Rightarrow 4 \sin^2(\theta) + 2 \sin(\theta) - 1 = 0$$

$$\sin(\theta) = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$= \frac{-1 \pm \sqrt{5}}{4}$$

$$\frac{-1 - \sqrt{5}}{4} \quad \frac{\sqrt{5} - 1}{4}$$

X ✓

2 क्यों

हमें बखुबी पता है कि \sin 0 से 45° में value 0 से $1/\sqrt{2}$ के बीच में होता है इसलिए उ वाला आंसर जो की finally negative quantity है इसलिए उसको consider नहीं करेंगे।

$$\therefore \sin(18^\circ) = \frac{(\sqrt{5} - 1)}{4}$$

(ii) $\cos(18^\circ)$

$$\because \sin^2 \theta + \cos^2 \theta = 1$$
$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \cos^2(18^\circ) = 1 - \sin^2(18^\circ)$$

$$\Rightarrow \cos^2 18^\circ = 1 - \frac{(\sqrt{5}-1)^2}{(4)^2}$$

$$\Rightarrow \cos^2 18^\circ = \frac{16 - (5 + 1 - 2\sqrt{5})}{16}$$

$$\Rightarrow \cos^2 18^\circ = \frac{16 - 6 + 2\sqrt{5}}{16}$$

$$\Rightarrow \cos^2 18^\circ = \frac{10 + 2\sqrt{5}}{16}$$

$$\Rightarrow \cos(18^\circ) = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$$

→ we want answer इसलिए neglect किए
वर्गों की $\cos 0$ से 45° में उसकी value
1 से $1/\sqrt{2}$ के बीच में होती है।

$$\therefore \cos(18^\circ) = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$$

iii

$$\cos(36^\circ)$$

$$= \cos(36^\circ)$$

$$= \cos 2(18^\circ)$$

$$= 1 - 2 \sin^2(18^\circ)$$

$$\left\{ \because \cos(2\theta) = 1 - 2 \sin^2(\theta) \right\}$$

$$= \left\{ 1 - 2 \cdot \frac{(\sqrt{5} - 1)^2}{(4)^2} \right\}$$

$$= \left\{ 1 - 2 \cdot \frac{(\sqrt{5} - 1)^2}{16} \right\}$$

$$= \left\{ 1 - \frac{(\sqrt{5} - 1)^2}{8} \right\}$$

$$= \left\{ \frac{8 - (5 + 1 - 2\sqrt{5})}{8} \right\}$$

$$= \left\{ \frac{8 - 6 + 2\sqrt{5}}{8} \right\}$$

$$= \left\{ \frac{2 + 2\sqrt{5}}{8} \right\} = \frac{\sqrt{5} + 1}{4}$$

$$\therefore \cos(36^\circ) = \frac{\sqrt{5}+1}{4}$$

(iv) $\sin(36^\circ)$

~~$= \sin 2(18^\circ)$~~

$$\left\{ \begin{array}{l} \because \sin^2(\theta) + \cos^2(\theta) = 1 \\ \sin^2 \theta = 1 - \cos^2 \theta \\ \sin \theta = \sqrt{1 - \cos^2(\theta)} \end{array} \right.$$

$$\Rightarrow \sin(36^\circ) = \sqrt{1 - \cos^2 36^\circ}$$

$$\Rightarrow \sin(36^\circ) = \left\{ 1 - \frac{(\sqrt{5}+1)^2}{16} \right\}^{1/2}$$

$$\Rightarrow \sin(36^\circ) = \left\{ \frac{16 - 5 - 1 - 2\sqrt{5}}{16} \right\}^{1/2}$$

$$\Rightarrow \sin(36^\circ) = \left\{ \frac{10 - 2\sqrt{5}}{16} \right\}^{1/2}$$

$$\Rightarrow \sin(36^\circ) = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$$

$$\therefore \sin(36^\circ) = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

(V) $\sin(72^\circ)$

Ans:
$$\begin{aligned} &= \sin(90^\circ - 18^\circ) \\ &= \cos(18^\circ) \\ &= \frac{\sqrt{10+2\sqrt{5}}}{4} \end{aligned}$$

$$\therefore \sin(72^\circ) = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

(Vi) $\cos(72^\circ)$

Ans:
$$\begin{aligned} &= \cos(90^\circ - 18^\circ) \\ &= \sin 18^\circ \\ &= \frac{(\sqrt{5}-1)}{4} \end{aligned}$$

$$\therefore \cos(72^\circ) = \frac{\sqrt{5}-1}{4}$$

(P.T.O.)

(vii) $\sin(54^\circ)$

$$\begin{aligned}
 &= \sin(90^\circ - 36^\circ) \\
 &= \cos(36^\circ) \\
 &= \frac{\sqrt{5}+1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \sin(54^\circ) &= \frac{\sqrt{5}+1}{4}
 \end{aligned}$$

(viii) $\cos(54^\circ)$

$$\begin{aligned}
 &= \cos(90^\circ - 36^\circ) \\
 &= \sin(36^\circ) \\
 &= \frac{\sqrt{10-2\sqrt{5}}}{4}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \cos(54^\circ) &= \frac{\sqrt{10-2\sqrt{5}}}{4}
 \end{aligned}$$

Q. Prove that : —

$$\sin\left(\frac{\pi}{10}\right) + \sin\left(\frac{13\pi}{10}\right) = -\frac{1}{2}$$

Ans: L.H.S.

$$= \sin\left(\frac{\pi}{10}\right) + \sin\left(\frac{13\pi}{10}\right)$$

$$= \sin\left(\frac{\pi}{10}\right) + \sin\left(\pi + \frac{3\pi}{10}\right)$$

$$= \sin\left(\frac{\pi}{10}\right) - \sin\left(\frac{3\pi}{10}\right) \left\{ \because \sin(\pi + \theta) = -\sin\theta \right\}$$

$$= \sin 18^\circ - \sin 54^\circ$$

$$= \sin 18^\circ - \sin(90^\circ - 36^\circ)$$

$$= \sin 18^\circ - \cos 36^\circ$$

$$= \left\{ \frac{(\sqrt{5}-1)}{4} - \frac{(\sqrt{5}+1)}{4} \right\}$$

$$= \frac{\sqrt{5}-1 - \sqrt{5}-1}{4}$$

$$= \frac{-2}{4} = -\frac{1}{2} \quad \text{--- P.H.S.}$$

Q. Prove that :-

$$\cos^2(48^\circ) - \sin^2(12^\circ) = \frac{(\sqrt{5}+1)}{8}$$

Ans:

L.H.S.

$$= \cos^2(48^\circ) - \sin^2(12^\circ)$$

$$= \frac{1}{2} \cdot (2 \cdot \cos^2(48^\circ) - 2 \cdot \sin^2(12^\circ))$$

$$= \frac{1}{2} \{ (1 + \cos 96^\circ) - (1 - \cos 24^\circ) \}$$

$$= \frac{1}{2} \{ \cos(96^\circ) + \cos(24^\circ) \}$$

$$\left\{ \begin{array}{l} \because 1 + \cos(2\theta) = 2 \cdot \cos^2(\theta) \\ 1 - \cos(2\theta) = 2 \cdot \sin^2(\theta) \\ \cos C + \cos D = 2 \cdot \cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right) \end{array} \right.$$

$$= \frac{1}{2} \left\{ 2 \cdot \cos\left(\frac{96^\circ + 24^\circ}{2}\right) \cdot \cos\left(\frac{96^\circ - 24^\circ}{2}\right) \right\}$$

$$= \frac{1}{2} \times 2 \cdot \cos(60^\circ) \cdot \cos(36^\circ)$$

$$= \frac{1}{2} \times 2 \times \frac{1}{2} \times \frac{(\sqrt{5} + 1)}{4}$$

$$= \frac{\sqrt{5} + 1}{8} = \text{R.H.S.} \quad \text{proven}$$

Q. Prove that :-

$$\sin^2(72^\circ) - \sin^2(60^\circ) = \frac{(\sqrt{5} - 1)}{8}$$

Ans:

L.H.S.

$$= \sin^2(72^\circ) - \sin^2(60^\circ)$$

$$= \frac{1}{2} (2 \cdot \sin^2(72^\circ) - 2 \cdot \sin^2(60^\circ))$$

$$= \frac{1}{2} \{ (1 - \cos(144^\circ)) - (1 - \cos(120^\circ)) \}$$

$$\left\{ \because 2 \cdot \sin^2(\theta) = 1 - \cos(2\theta) \right\}$$

$$= \frac{1}{2} (\cos(120^\circ) - \cos(144^\circ))$$

$$= \frac{1}{2} \left(-\frac{1}{2} - \cos(144^\circ) \right) \left[\because \cos(120^\circ) = -\frac{1}{2} \right]$$

$$= -\frac{1}{4} - \frac{1}{2} \cdot \cos(144^\circ)$$

$$= -\frac{1}{4} - \frac{1}{2} \cdot \cos(180^\circ - 36^\circ)$$

$$= -\frac{1}{4} + \frac{1}{2} \cdot \cos(36^\circ)$$

$$= -\frac{1}{4} + \frac{1}{2} \left(\frac{\sqrt{5}+1}{4} \right)$$

$$\left\{ \because \cos 36^\circ = \frac{(\sqrt{5}+1)}{4} \right\}$$

$$= \frac{(\sqrt{5}-1)}{8}$$

= R.H.S. proved //

Q. Prove that : —

$$\cos 6^\circ \cdot \cos 42^\circ \cdot \cos 66^\circ \cdot \cos 78^\circ = \frac{1}{16}$$

Ans:

L.H.S.

$$= \cos 6^\circ \cdot \cos 42^\circ \cdot \cos 66^\circ \cdot \cos 78^\circ$$

$$= \frac{1}{2} (2 \cdot \cos 66^\circ \cdot \cos 6^\circ) \cdot \frac{1}{2} (2 \cdot \cos 78^\circ \cdot \cos 42^\circ)$$

$$= \frac{1}{4} [\cos(66^\circ + 6^\circ) + \cos(66^\circ - 6^\circ)] \times [\cos(78^\circ + 42^\circ) + \cos(78^\circ - 42^\circ)]$$

$$= \frac{1}{4} (\cos 72^\circ + \cos 60^\circ) \cdot (\cos 120^\circ + \cos 36^\circ)$$

$$= \frac{1}{4} \left(\sin 18^\circ + \frac{1}{2} \right) \cdot \left(-\frac{1}{2} + \cos 36^\circ \right)$$

$$\left\{ \because \cos 72^\circ = \cos(90^\circ - 18^\circ) = \sin 18^\circ \right\}$$

$$\left\{ \begin{array}{l} \sin(18^\circ) = \frac{(\sqrt{5}-1)}{4} \text{ \& } \cos(36^\circ) = \frac{(\sqrt{5}+1)}{4} \end{array} \right\}$$

$$= \frac{1}{4} \left[\frac{(\sqrt{5}-1)}{4} + \frac{1}{2} \right] \left[-\frac{1}{2} + \frac{\sqrt{5}+1}{4} \right]$$

$$= \frac{1}{4} \times \frac{\sqrt{5}+1}{4} \times \frac{\sqrt{5}-1}{4}$$

$$= \frac{5-1}{64} = \frac{4}{64} = \frac{1}{16} = \text{R.H.S. proved}$$

Q. Prove that: —

$$\sin\left(\frac{\pi}{5}\right) \cdot \sin\left(\frac{2\pi}{5}\right) \cdot \sin\left(\frac{3\pi}{5}\right) \cdot \sin\left(\frac{4\pi}{5}\right) = \frac{5}{16}$$

Ans: L.H.S.

$$= \sin\left(\frac{\pi}{5}\right) \cdot \sin\left(\frac{2\pi}{5}\right) \cdot \sin\left(\frac{3\pi}{5}\right) \cdot \sin\left(\frac{4\pi}{5}\right)$$

$$= \sin\left(\frac{\pi}{5}\right) \cdot \sin\left(\frac{2\pi}{5}\right) \cdot \sin\left(\pi - \frac{2\pi}{5}\right) \cdot \sin\left(\pi - \frac{\pi}{5}\right)$$

$$= \sin^2\left(\frac{\pi}{5}\right) \cdot \sin^2\left(\frac{2\pi}{5}\right)$$

$$= \sin^2(36^\circ) \times \sin^2(72^\circ)$$

$$= (\sin(36^\circ))^2 \times (\cos(18^\circ))^2 \left\{ \begin{array}{l} \because \sin(72^\circ) \\ = \sin(90^\circ - 18^\circ) = \cos(18^\circ) \end{array} \right\}$$

$$= \frac{(10 - 2\sqrt{5})}{16} \times \frac{(10 + 2\sqrt{5})}{16} = \frac{100 - 20}{16 \times 16}$$

$$= \frac{80}{16 \times 16} = \frac{5}{16} = \text{R.H.S.}$$

★★ find the value of :-

(i) $\sin\left(22\frac{1}{2}^\circ\right)$

Ans: $\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$

$$\Rightarrow \sin^2\left(22\frac{1}{2}^\circ\right) = \frac{1 - \cos(45^\circ)}{2}$$

$$= \frac{\left(1 - \frac{1}{\sqrt{2}}\right)}{2} = \frac{\sqrt{2} - 1}{2 \cdot \sqrt{2}}$$

$$\Rightarrow \sin\left(22\frac{1}{2}^\circ\right) = \sqrt{\frac{(\sqrt{2} - 1)}{2 \cdot \sqrt{2}}}$$

$$\therefore \sin\left(22\frac{1}{2}^\circ\right) = \sqrt{\frac{(\sqrt{2} - 1)}{2 \cdot \sqrt{2}}}$$

(ii) $\cos\left(22\frac{1}{2}^\circ\right)$

Ans: $\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$

$$\Rightarrow \cos^2\left(22\frac{1}{2}^\circ\right) = \frac{1 + \cos(45^\circ)}{2}$$

(P.T.O.)

$$= \frac{\left(1 + \frac{1}{\sqrt{2}}\right)}{2} = \frac{\sqrt{2} + 1}{2\sqrt{2}}$$

$$\Rightarrow \cos\left(22\frac{1}{2}^\circ\right) = \sqrt{\frac{\sqrt{2} + 1}{2\sqrt{2}}}$$

$$\therefore \cos\left(22\frac{1}{2}^\circ\right) = \sqrt{\frac{\sqrt{2} + 1}{2\sqrt{2}}}$$

$$(iii) \tan\left(22\frac{1}{2}^\circ\right)$$

Ans: $\tan^2\left(22\frac{1}{2}^\circ\right) = \frac{\sin^2\left(22\frac{1}{2}^\circ\right)}{\cos^2\left(22\frac{1}{2}^\circ\right)}$

$$= \frac{\sqrt{2} - 1}{2\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{2} + 1}$$

$$= \frac{(\sqrt{2} - 1)^2}{(\sqrt{2})^2 - (1)^2} = \frac{(\sqrt{2} - 1)^2}{2 - 1}$$

$$= (\sqrt{2} - 1)^2$$

$$\Rightarrow \tan\left(22\frac{1}{2}^\circ\right) = (\sqrt{2} - 1)$$

$$\therefore \tan\left(22\frac{1}{2}^\circ\right) = \sqrt{2} - 1$$

Q. Prove that: —

$$\frac{\cos(2x)}{\cos(x) - \sin(x)} = \cos(x) + \sin(x)$$

Ans:

L.H.S.

$$= \frac{\cos(2x)}{\cos(x) - \sin(x)}$$

$$\left\{ \because \cos(2x) = \cos^2(x) - \sin^2(x) \right\}$$

$$= \frac{\cos^2(x) - \sin^2(x)}{\cos(x) - \sin(x)}$$

$$= \frac{(\cos(x) + \sin(x)) \cdot (\cos(x) - \sin(x))}{(\cos(x) - \sin(x))}$$

$$= \cos(x) + \sin(x) = R.H.S.$$

Q. Prove that: —

$$\frac{\sin(2x)}{1 + \cos(2x)} = \tan(x)$$

Ans:

L.H.S.

$$= \frac{\sin(2x)}{1 + \cos(2x)}$$

$$\text{Q.} \quad \frac{2 \sin(x) \cos(x)}{2 \cos^2(x)}$$

$$= \frac{\sin(x)}{\cos(x)} = \tan(x) = \text{R.H.S.}$$

Q. Prove that :-

$$\frac{\sin(2x)}{1 - \cos(2x)} = \cot(x)$$

Ans: L.H.S.

$$= \frac{\sin(2x)}{1 - \cos(2x)} = \frac{2 \sin(x) \cos(x)}{2 \sin^2(x)}$$

$$= \frac{\cos(x)}{\sin(x)} = \cot(x) = \text{R.H.S.}$$

Q. Prove that :-

$$\frac{\tan(2x)}{1 + \sec(2x)} = \tan(x)$$

Ans: L.H.S.

$$= \frac{\tan(2x)}{1 + \sec(2x)}$$

$$= \frac{\frac{\sin(2x)}{\cos(2x)}}{1 + \frac{1}{\cos(2x)}} = \frac{\frac{\sin(2x)}{\cos(2x)}}{\frac{\cos(2x) + 1}{\cos(2x)}}$$

$$= \frac{\sin(2x)}{1 + \cos(2x)} = \frac{2 \sin(x) \cdot \cos(x)}{2 \cos^2(x)}$$

$$= \frac{\sin(x)}{\cos(x)} = \tan(x) = \text{R.H.S.}$$

Q. Prove that:—
 $\sin(2x) \cdot (\tan(x) + \cot(x)) = 2$

Ans: L.H.S.

$$= \sin(2x) \cdot (\tan(x) + \cot(x))$$

$$= 2 \sin(x) \cdot \cos(x) \left(\frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)} \right)$$

$$= 2 \sin(x) \cdot \cos(x) \left(\frac{\sin^2(x) + \cos^2(x)}{\sin(x) \cdot \cos(x)} \right)$$

$$= 2 \times 1 = 2 = \text{R.H.S.}$$

Q. Prove that:—

$$\operatorname{cosec}(2x) + \cot(2x) = \cot(x)$$

Ans: L.H.S.

$$= \frac{1}{\sin(2x)} + \frac{\cos(2x)}{\sin(2x)}$$

$$= \frac{1 + \cos(2x)}{\sin(2x)}$$

$$= \frac{2 \cos^2(x)}{2 \sin(x) \cdot \cos(x)} = \frac{\cos(x)}{\sin(x)} = \cot(x) = \text{R.H.S.}$$

Q. Prove that :-

$$\cos(2x) + 2 \cdot \sin^2(x) = 1.$$

Ans: L.H.S.

$$= \cos(2x) + 2 \cdot \sin^2(x)$$

$$= 1 - 2 \cdot \sin^2(x) + 2 \cdot \sin^2(x)$$

$$= 1 \quad = R.H.S.$$

Q. Prove that :-

$$(\sin(x) - \cos(x))^2 = 1 - \sin(2x)$$

Ans: L.H.S.

$$= (\sin(x) - \cos(x))^2$$

$$= \sin^2(x) + \cos^2(x) - 2 \cdot \sin(x) \cdot \cos(x)$$

$$= 1 - \sin(2x)$$

$$= R.H.S.$$

Q. Prove that :-

$$\cot(x) - 2 \cdot \cot(2x) = \tan(x)$$

Ans: L.H.S.

$$= \cot(x) - 2 \cdot \cot(2x)$$

$$= \frac{1}{\tan(x)} - 2 \cdot \frac{1}{\tan(2x)}$$

$$= \frac{\cos(x)}{\sin(x)} - 2 \cdot \frac{\cos(2x)}{\sin(2x)}$$

$$= \frac{1}{\tan(x)} - \frac{2(1 - \tan^2(x))}{2 \tan(x)} = \frac{1 - 1 + \tan^2(x)}{\tan(x)} = \tan(x) = R.H.S.$$

Q. Prove that :-

$$(\cos^4(x) + \sin^4(x)) = \frac{1}{2} \cdot (2 - \sin^2(2x))$$

Ans:

L.H.S.

$$= (\cos^2(x))^2 + (\sin^2(x))^2$$

$$\left\{ \because a^2 + b^2 = (a+b)^2 - 2a \cdot b \right\}$$

$$= (\sin^2(x) + \cos^2(x))^2 - 2 \cdot \sin^2(x) \cdot \cos^2(x)$$

$$= 1 - 2 \cdot \sin^2(x) \cdot \cos^2(x)$$

$$= 1 - \frac{1 \cdot 2 \cdot 2 \sin^2(x) \cdot \cos^2(x)}{2}$$

$$= 1 - \frac{4 \sin^2(x) \cos^2(x)}{2}$$

$$= 1 - \frac{\sin^2(2x)}{2}$$

$$= \frac{1}{2} \cdot (2 - \sin^2(2x)) = R.H.S.$$

Q. Prove that :-

$$\frac{\cos^3(x) - \sin^3(x)}{\cos(x) - \sin(x)} = \frac{1}{2} (2 + \sin(2x))$$

(P.T.O.)

Ans:

L.H.S.

$$= \frac{\cos^3(x) - \sin^3(x)}{\cos(x) - \sin(x)}$$

$$= \frac{(\cos(x) - \sin(x)) \cdot (\cos^2(x) + \cos(x) \cdot \sin(x) + \sin^2(x))}{(\cos(x) - \sin(x))}$$

$$= \cos^2(x) + \cos(x) \cdot \sin(x) + \sin^2(x)$$

$$= (\cos^2(x) + \sin^2(x)) + \cos(x) \cdot \sin(x)$$

$$= 1 + \cos(x) \cdot \sin(x)$$

$$= \frac{1}{2} (2 + 2 \cdot \cos(x) \cdot \sin(x))$$

$$= \frac{1}{2} (2 + \sin(2x)) \quad \text{proved} \checkmark$$

Q. To prove that:—

$$\frac{1 - \cos(2x) + \sin(x)}{\sin(2x) + \cos(x)} = \tan(x)$$

Ans:

L.H.S.

$$= \frac{1 - \cos(2x) + \sin(x)}{\sin(2x) + \cos(x)}$$

$$= \frac{(1 - \cos(2x)) + \sin(x)}{\sin(2x) + \cos(x)}$$

$$= \frac{2 \cdot \sin^2(x) + \sin(x)}{2 \cos(x) \cdot \sin(x) + \cos(x)}$$

$$= \frac{\sin(x) (2 \sin(x) + 1)}{\cos(x) (2 \cdot \frac{\cos(x)}{\sin(x)} + 1)}$$

$$= \tan(x) = R.H.S.$$

Q. Prove that : —

$$\cos(x) \cdot \cos(2x) \cdot \cos(4x) \cdot \cos(8x) = \frac{\sin(16x)}{16 \sin(x)}$$

Ans: L.H.S.

$$= \cos(x) \cdot \cos(2x) \cdot \cos(4x) \cdot \cos(8x)$$

$$= \frac{1}{2 \sin(x)} \left[2 \cdot \sin(x) \cdot \cos(x) \cdot \cos(2x) \cdot \cos(4x) \cdot \cos(8x) \right]$$

$$= \frac{1}{2 \sin(x)} \left[(2 \sin(x) \cdot \cos(x)) \cdot \cos(2x) \cdot \cos(4x) \cdot \cos(8x) \right]$$

$$= \frac{1}{2 \sin(x)} \left[\sin(2x) \cdot \cos(2x) \cdot \cos(4x) \cdot \cos(8x) \right]$$

$$= \frac{1}{2 \times 2 \sin(x)} \left[[2 \sin(2x) \cdot \cos(2x)] \cdot \cos(4x) \cdot \cos(8x) \right]$$

$$= \frac{1}{4 \sin(x)} \left[\sin(4x) \cdot \cos(4x) \cdot \cos(8x) \right]$$

$$= \frac{1}{2 \times 4 \sin(x)} \left[2 \sin(4x) \cdot \cos(4x) \cdot \cos(8x) \right]$$

$$= \frac{1}{8 \cdot \sin(x)} \cdot [\sin(8x) \cdot \cos(8x)]$$

$$= \frac{1}{2 \times 8 \sin(x)} \cdot [2 \sin(8x) \cdot \cos(8x)]$$

$$= \frac{1}{16 \cdot \sin(x)} \cdot [\sin(16x)]$$

$$= \frac{\sin(16x)}{16 \cdot \sin(x)} = \text{R.H.S.} \quad \text{Proved}$$

Q. Prove that:—

$$2 \cdot \sin\left(22\frac{1}{2}^\circ\right) \cdot \cos\left(22\frac{1}{2}^\circ\right) = \frac{1}{\sqrt{2}}$$

Ans: L.H.S.

$$= 2 \cdot \sin\left(22\frac{1}{2}^\circ\right) \cdot \cos\left(22\frac{1}{2}^\circ\right)$$

$$= \sin 2\left(\frac{45}{2}\right) = \sin 45^\circ = \frac{1}{\sqrt{2}} = \text{R.H.S.}$$

Q. Prove that:—

$$8 \cdot \cos^3(20^\circ) - 6 \cdot \cos(20^\circ) = 1.$$

Ans: $= 2(4 \cdot \cos^3(20^\circ) - 3 \cdot \cos(20^\circ))$

$$= 2 [\cos 3(20^\circ)]$$

$$= 2 [\cos 60^\circ]$$

$$= 2 \times \frac{1}{2} = 1$$

$$= \text{R.H.S.}$$

Proved

Q. Prove that :-

$$3 \cdot \sin(40^\circ) - 4 \cdot \sin^3 40^\circ = \frac{\sqrt{3}}{2}$$

Ans:

$$\begin{aligned} & \text{L.H.S.} \\ &= 3 \cdot \sin(40^\circ) - 4 \cdot \sin^3(40^\circ) \\ &= \sin 3(40^\circ) \\ &= \sin(120^\circ) \\ &= \sin(180^\circ - 60^\circ) \\ &= \sin 60^\circ \\ &= \frac{\sqrt{3}}{2} = \text{R.H.S.} \quad \text{Proved} \end{aligned}$$

Q. Prove that :-

$$\sin^2(24^\circ) - \sin^2(6^\circ) = \frac{(\sqrt{5}-1)}{8}$$

Ans:

$$\begin{aligned} & \text{L.H.S.} \\ &= \sin^2(24^\circ) - \sin^2(6^\circ) \\ &= \sin(24^\circ + 6^\circ) \cdot \sin(24^\circ - 6^\circ) \\ &= \sin(30^\circ) \cdot \sin(18^\circ) \\ &= \frac{1}{2} \times \frac{\sqrt{5}-1}{4} = \frac{\sqrt{5}-1}{8} \\ &= \text{R.H.S.} \end{aligned}$$

Q. Prove that :-

$$\tan 6^\circ \cdot \tan 42^\circ \cdot \tan 66^\circ \cdot \tan 78^\circ = 1$$

(P.T.O.)

Ans: L.H.S.

$$\begin{aligned}
 &= \tan(6^\circ) \cdot \tan(42^\circ) \cdot \tan(66^\circ) \cdot \tan(78^\circ) \\
 &= \frac{\tan(6^\circ) \cdot \tan(42^\circ) \cdot \tan(66^\circ) \cdot \tan(78^\circ)}{\tan(54^\circ) \cdot \tan(18^\circ)} \\
 &= \frac{(\tan(6^\circ) \cdot \tan(54^\circ) \cdot \tan(66^\circ)) (\tan(18^\circ) \cdot \tan(42^\circ) \cdot \tan(72^\circ))}{\tan(54^\circ) \cdot \tan(18^\circ)} \\
 &= \frac{[\tan 3(6^\circ)] \cdot [\tan 3(18^\circ)]}{\tan(54^\circ) \cdot \tan(18^\circ)} \\
 &= \frac{\cancel{\tan(18^\circ)} \cdot \cancel{\tan(54^\circ)}}{\cancel{\tan(54^\circ)} \cdot \cancel{\tan(18^\circ)}} = 1 = \text{R.H.S.} \\
 &\quad \text{Proved}
 \end{aligned}$$

Q. If $\tan(x) = -\frac{4}{3}$ and $\frac{\pi}{2} < x < \pi$, find the values of :-

(i) $\sin\left(\frac{x}{2}\right)$ (ii) $\cos\left(\frac{x}{2}\right)$ (iii) $\tan\left(\frac{x}{2}\right)$

Ans: $\because x$ lies in quadrant II, we have $\cos(x) < 0$

$$\because \tan(x) = -\frac{4}{3}$$

$$\Rightarrow \sec^2(x) = (1 + \tan^2(x))$$

$$= 1 + \frac{16}{9}$$

$$= \frac{25}{9}$$

$$\Rightarrow \cos^2(x) = \frac{1}{\sec^2(x)} = \frac{9}{25}$$

$$\Rightarrow \cos(x) = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

क्योंकि $\cos(x)$ IInd quadrant में है और हमें यह बहुत अच्छे से पता है कि IInd quadrant में \cos -ve है।

$$\text{also, } \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

↳ this implies that $\frac{x}{2}$ lies in Quadrant I.

$$\Rightarrow \sin\left(\frac{x}{2}\right) > 0 \text{ and } \cos\left(\frac{x}{2}\right) > 0.$$

$$(i) \quad 2 \sin^2\left(\frac{x}{2}\right) = (1 - \cos(x))$$

$$2 \sin^2\left(\frac{x}{2}\right) = 1 - \left(-\frac{3}{5}\right)$$

$$2 \sin^2\left(\frac{x}{2}\right) = 1 + \frac{3}{5} = \frac{8}{5}$$

$$\Rightarrow \sin^2\left(\frac{x}{2}\right) = \frac{4}{5}$$

$$\Rightarrow \sin\left(\frac{x}{2}\right) = +\sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

क्योंकि $\frac{\pi}{2} < x < \pi \Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$ इसलिए positive sine है।

$$(ii.) \cos\left(\frac{x}{2}\right)$$

$$2 \cos^2\left(\frac{x}{2}\right) = (1 + \cos(x))$$

$$= \left(1 - \frac{3}{5}\right) = \frac{2}{5}$$

$$\cos^2\left(\frac{x}{2}\right) = \frac{2}{5 \times 2} = \frac{1}{5}$$

$$\cos\left(\frac{x}{2}\right) = +\sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} \quad \left\{ \because \cos\left(\frac{x}{2}\right) > 0 \right\}$$

$$(iii.) \tan\left(\frac{x}{2}\right)$$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \left(\frac{2}{\sqrt{5}} \times \sqrt{5}\right) = 2$$

Q. If $\cos(x) = -\frac{1}{3}$ and x lies in Quadrant III, find the values of :-

$$(i.) \sin\left(\frac{x}{2}\right) \quad (ii.) \cos\left(\frac{x}{2}\right) \quad (iii.) \tan\left(\frac{x}{2}\right)$$

Ans: $\because x$ lies in Quadrant III, we have

$$\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

$\Rightarrow \left(\frac{x}{2}\right)$ lies in Quadrant IInd

$$\Rightarrow \sin\left(\frac{x}{2}\right) > 0 \text{ and } \cos\left(\frac{x}{2}\right) < 0$$

$$(i) 2 \sin^2\left(\frac{\pi}{2}\right)$$

$$= 1 - \cos(\pi)$$

$$= \left(1 + \frac{1}{3}\right) = \frac{4}{3}$$

$$\Rightarrow 2 \sin^2\left(\frac{\pi}{2}\right) = \frac{4}{3}$$

$$\Rightarrow \sin^2\left(\frac{\pi}{2}\right) = \frac{4^2}{2 \times 3}$$

$$\Rightarrow \sin\left(\frac{\pi}{2}\right) = +\sqrt{\frac{2}{3}}$$

$$\Rightarrow \sin\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

→ क्योंकि $\sin\left(\frac{\pi}{2}\right)$ IInd quadrant में है और हमें \sin की मान $\frac{\pi}{2}$ की IInd quadrant में $\sin(\pi)$ +ve होता है।

$$(ii) 2 \cos^2\left(\frac{\pi}{2}\right) = (1 + \cos(\pi))$$

$$= \left(1 - \frac{1}{3}\right) = \frac{2}{3}$$

$$\Rightarrow \cos^2\left(\frac{\pi}{2}\right) = \frac{1}{3}$$

$$\Rightarrow \cos\left(\frac{\pi}{2}\right) = -\frac{1}{\sqrt{3}}$$

→ क्योंकि $\cos\left(\frac{\pi}{2}\right)$ IInd quadrant में है और हमें \cos की मान $\frac{\pi}{2}$ की IInd quadrant में $\cos(\pi)$ -ve होता है।

(iii.) $\tan\left(\frac{\pi}{2}\right)$

$$= \frac{\sin\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right)} = \frac{\sqrt{6}}{3} \times \frac{3}{-\sqrt{3}} = -\sqrt{2}.$$

Q. Prove that :-

$$\frac{1+\cos(x)}{1-\cos(x)} = (\operatorname{cosec}(x) + \cot(x))^2$$

Ans:

L.H.S.

$$= \frac{1+\cos(x)}{1-\cos(x)} = \frac{x \cdot \cos^2\left(\frac{x}{2}\right)}{x \cdot \sin^2\left(\frac{x}{2}\right)} = \cot^2\left(\frac{x}{2}\right)$$

R.H.S.

$$= (\operatorname{cosec}(x) + \cot(x))^2$$

$$= \left(\frac{1}{\sin(x)} + \frac{\cos(x)}{\sin(x)} \right)^2$$

$$= \left(\frac{1+\cos(x)}{\sin(x)} \right)^2$$

$$= \left\{ \frac{x \cdot \cos^2\left(\frac{x}{2}\right)}{x \cdot \sin\left(\frac{x}{2}\right) \cdot \cos\left(\frac{x}{2}\right)} \right\}^2$$

$$= \left\{ \frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)} \right\}^2 = \cot^2\left(\frac{x}{2}\right)$$

$\therefore L.H.S = R.H.S$ *proven*

Q. Prove that:—

$$\frac{\cos(x)}{(1 - \sin(x))} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

Ans:

L.H.S.

$$= \frac{(\cos^2(\frac{x}{2}) - \sin^2(\frac{x}{2}))}{(\cos^2(\frac{x}{2}) + \sin^2(\frac{x}{2}) - 2 \cdot \sin(\frac{x}{2}) \cdot \cos(\frac{x}{2}))}$$

$$= \frac{(\cancel{\cos(\frac{x}{2}) - \sin(\frac{x}{2})}) \cdot (\cos(\frac{x}{2}) + \sin(\frac{x}{2}))}{(\cos(\frac{x}{2}) - \sin(\frac{x}{2}))^2}$$

$$= \frac{\cos(\frac{x}{2}) + \sin(\frac{x}{2})}{\cos(\frac{x}{2}) - \sin(\frac{x}{2})}$$

$$= \frac{1 + \tan(\frac{x}{2})}{1 - \tan(\frac{x}{2})}$$

$$\left\{ \begin{array}{l} \text{dividing num.} \\ \& \text{den. by} \\ \cos(\frac{x}{2}) \end{array} \right.$$

$$= \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \text{R.H.S.}$$

Q. Prove that:-

$$\cot\left(\frac{x}{2}\right) - \tan\left(\frac{x}{2}\right) = 2 \cdot \cot(x)$$

Ans: L.H.S.

$$= \cot\left(\frac{x}{2}\right) - \tan\left(\frac{x}{2}\right)$$

$$= \frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)} - \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$$

$$= \frac{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) \cdot \cos\left(\frac{x}{2}\right)}$$

$$= \frac{2 \cos(x)}{2 \cdot \sin\left(\frac{x}{2}\right) \cdot \cos\left(\frac{x}{2}\right)}$$

$$= \frac{2 \cdot \cos(x)}{\sin(x)}$$

$$= 2 \cdot \cot(x)$$

$$= \text{R.H.S.}$$

proven

Q. Prove that:-

$$\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \tan(x) + \sec(x)$$

Ans:

L.H.S.

$$= \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

$$= \frac{\tan\left(\frac{\pi}{4}\right) + \tan\left(\frac{x}{2}\right)}{1 - \tan\left(\frac{\pi}{4}\right) \cdot \tan\left(\frac{x}{2}\right)}$$

$$= \frac{1 + \tan(x/2)}{1 - \tan(x/2)}$$

$$= \frac{1 + \frac{\sin(x/2)}{\cos(x/2)}}{1 - \frac{\sin(x/2)}{\cos(x/2)}}$$

$$= \frac{1 + \frac{\sin(x/2)}{\cos(x/2)}}{1 - \frac{\sin(x/2)}{\cos(x/2)}}$$

$$= \frac{\cos(x/2) + \sin(x/2)}{\cos(x/2) - \sin(x/2)}$$

$$= \frac{\cos(x/2) + \sin(x/2)}{\cos(x/2) - \sin(x/2)} \times \frac{\cos(x/2) + \sin(x/2)}{\cos(x/2) + \sin(x/2)}$$

$$= \frac{(\cos(\frac{x}{2}) + \sin(\frac{x}{2}))^2}{\cos^2(\frac{x}{2}) - \sin^2(\frac{x}{2})}$$

$$\cos^2(\frac{x}{2}) + \sin^2(\frac{x}{2}) + 2 \cdot \sin(\frac{x}{2}) \cos(\frac{x}{2})$$

$$= \frac{\cos^2(\frac{x}{2}) + \sin^2(\frac{x}{2}) + 2 \cdot \sin(\frac{x}{2}) \cos(\frac{x}{2})}{\cos(x)}$$

$$= \frac{1 + \sin(x)}{\cos(x)}$$

$$= \frac{1}{\cos(x)} + \frac{\sin(x)}{\cos(x)}$$

$$= \sec(x) + \tan(x)$$

$$= \text{R.H.S.}$$

Proved

Q. Prove that :-

$$\sqrt{\frac{1 + \sin(x)}{1 - \sin(x)}} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

Ans: L.H.S.

$$= \sqrt{\frac{1 + \sin(x)}{1 - \sin(x)}} \times \frac{1 + \sin(x)}{1 + \sin(x)}$$

$$= \sqrt{\frac{(1 + \sin(x))^2}{1 - \sin^2(x)}}$$

$$= \frac{1 + \sin(x)}{\cos(x)} = \frac{1 + 2 \cdot \sin(\frac{x}{2}) \cdot \cos(\frac{x}{2})}{\cos(x)}$$

$$= \frac{\cos^2\left(\frac{x}{2}\right) + \sin^2\left(\frac{x}{2}\right) + 2 \cdot \cos\left(\frac{x}{2}\right) \sin\left(\frac{x}{2}\right)}{\cos(x)}$$

$$= \frac{\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)^2}{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}$$

$$= \frac{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}$$

$$= \frac{1 + \frac{\sin(x/2)}{\cos(x/2)}}{1 - \frac{\sin(x/2)}{\cos(x/2)}}$$

$$= \frac{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}$$

$$= \frac{1 + \tan\left(\frac{x}{2}\right)}{1 - \tan\left(\frac{x}{2}\right)}$$

$$= \frac{\tan(\pi/4) + \tan(x/2)}{1 - \tan(\pi/4) \cdot \tan(x/2)}$$

$$= \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) = \text{R.H.S. proved} //$$

Q. Prove that :-

$$\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) = 2 \cdot \sec(x)$$

Ans:

(L.H.S)

$$= \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$= \frac{\tan(\pi/4) + \tan(x/2)}{1 - \tan(\pi/4) \cdot \tan(x/2)} + \frac{\tan(\pi/4) - \tan(x/2)}{1 + \tan(\pi/4) \cdot \tan(x/2)}$$

$$= \frac{1 + \tan(x/2)}{1 - \tan(x/2)} + \frac{1 - \tan(x/2)}{1 + \tan(x/2)}$$

$$= \frac{1 + \frac{\sin(x/2)}{\cos(x/2)}}{1 - \frac{\sin(x/2)}{\cos(x/2)}} + \frac{1 - \frac{\sin(x/2)}{\cos(x/2)}}{1 + \frac{\sin(x/2)}{\cos(x/2)}}$$

$$= \frac{\frac{\sin(x/2) + \cos(x/2)}{\cos(x/2)}}{\frac{\cos(x/2) - \sin(x/2)}{\cos(x/2)}} + \frac{\frac{\cos(x/2) - \sin(x/2)}{\cos(x/2)}}{\frac{\cos(x/2) + \sin(x/2)}{\cos(x/2)}}$$

$$= \frac{\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)^2 + \left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right)^2}{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}$$

$$= \frac{2}{\cos(x)} = 2 \cdot \sec(x) = \text{R.H.S.} \quad \text{provel}$$

Q. Prove that :-

$$\frac{\sin(x)}{1 + \cos(x)} = \tan\left(\frac{x}{2}\right)$$

Ans:

L.H.S.

$$= \frac{\sin(x)}{1 + \cos(x)}$$

$$= \frac{2 \cdot \sin\left(\frac{x}{2}\right) \cdot \cancel{\cos\left(\frac{x}{2}\right)}}{2 \cdot \cos^2\left(\frac{x}{2}\right)}$$

$$= \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}$$

$$= \tan\left(\frac{x}{2}\right) = \text{R.H.S.}$$

provel

Q. If $A+B+C = \pi$, prove that

$$\sin(2A) + \sin(2B) + \sin(2C) = 4 \cdot \sin(A) \cdot \sin(B) \cdot \sin(C)$$

Ans:
L.H.S.

$$= (\sin(2A) + \sin(2B)) + \sin(2C)$$

$$= 2 \cdot \sin(A+B) \cdot \cos(A-B) + 2 \cdot \sin C \cdot \cos C$$

$$= 2 \cdot \sin(\pi - C) \cdot \cos(A-B) + 2 \cdot \sin C \cdot \cos C$$

$$\rightarrow \left\{ \begin{array}{l} \because A+B+C = \pi \\ (A+B) = \pi - C \end{array} \right\}$$

$$= 2 \cdot \sin(C) \cdot \cos(A-B) + 2 \cdot \sin C \cdot \cos C$$

$$= 2 \cdot \sin(C) [\cos(A-B) + \cos(C)]$$

$$= 2 \cdot \sin(C) [\cos(A-B) + \cos(\pi - (A+B))]$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ C = \pi - (A+B) \end{array} \right\} \leftarrow$$

$$= 2 \cdot \sin(C) [\cos(A-B) - \cos(A+B)]$$

$$= 2 \cdot \sin(C) [2 \cdot \sin(A) \cdot \sin(B)]$$

$$= 4 \cdot \sin(A) \cdot \sin(B) \cdot \sin(C)$$

= R.H.S.

Proved

Q. If $A+B+C = \pi$, prove that

$$\sin(2A) - \sin(2B) + \sin(2C) = 4 \cos(A) \sin(B) \cos(C)$$

Ans:

L.H.S.

$$= (\sin(2A) - \sin(2B)) + \sin(2C)$$

$$= 2 \cos(A+B) \sin(A-B) + 2 \sin C \cos C$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ A+B = (\pi - C) \end{array} \right\}$$

$$= 2 \cos(\pi - C) \sin(A-B) + 2 \sin C \cos C$$

$$= -2 \cos C \sin(A-B) + 2 \sin C \cos C$$

$$= -2 \cos C [\sin(A-B) - \sin C]$$

$$= -2 \cos C [\sin(A-B) - \sin\{\pi - (A+B)\}]$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ C = \pi - (A+B) \end{array} \right\}$$

$$= -2 \cos C [\sin(A-B) - \sin(A+B)]$$

$$= -2 \cos C [-2 \cos A \sin B]$$

$$= 4 \cos A \sin B \cos C$$

$$= R.H.S.$$

Proved

Q. If $A+B+C=\pi$, prove that

$$\cos(2A) + \cos(2B) + \cos(2C) = -1 - 4 \cdot \cos A \cdot \cos B \cdot \cos C$$

Ans: L.H.S.

$$\begin{aligned} &= \cos(2A) + \cos(2B) + \cos(2C) \\ &= 2 \cdot \cos(A+B) \cdot \cos(A-B) + 2 \cdot \cos^2(C) - 1 \\ &= 2 \cdot \cos(\pi-C) \cdot \cos(A-B) + 2 \cos^2(C) - 1 \\ &= -2 \cdot \cos C \cdot \cos(A-B) + 2 \cos^2(C) - 1 \\ &= -2 \cdot \cos C [\cos(A-B) - \cos(C)] - 1 \\ &= -1 - 2 \cdot \cos C [\cos(A-B) - \cos\{\pi-(A+B)\}] \\ &= -1 - 2 \cdot \cos C [\cos(A-B) + \cos(A+B)] \\ &= -1 - 2 \cdot \cos C [2 \cdot \cos A \cdot \cos B] \\ &= -1 - 4 \cdot \cos A \cdot \cos B \cdot \cos C = R.H.S. \end{aligned}$$

Q. If $A+B+C=\pi$, prove that

$$\cos(2A) + \cos(2B) - \cos(2C) = 1 - 4 \cdot \sin(A) \cdot \sin(B) \cdot \cos(C)$$

Ans: L.H.S.

$$\begin{aligned} &= (\cos(2A) + \cos(2B)) - \cos(2C) \\ &= 2 \cdot \cos(A+B) \cdot \cos(A-B) - \cos(2C) \\ &= 2 \cdot \cos(\pi-C) \cdot \cos(A-B) - 2 \cos^2(C) - 1 \\ &\quad \left\{ \begin{array}{l} \because A+B+C=\pi \\ A+B=\pi-C \end{array} \right\} \end{aligned}$$

$$\begin{aligned}
 &= -2 \cos(C) \cdot \cos(A-B) - 2 \cos^2(C) + 1 \\
 &= 1 - 2 \cos(C) [\cos(A-B) + \cos(C)] \\
 &= 1 - 2 \cos(C) [\cos(A-B) + \cos\{\pi - (A+B)\}]
 \end{aligned}$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ C = \pi - (A+B) \end{array} \right.$$

$$\begin{aligned}
 &= 1 - 2 \cos(C) [\cos(A-B) - \cos(A+B)] \\
 &= 1 - 2 \cos(C) [2 \sin(A) \sin(B)]
 \end{aligned}$$

$$\begin{aligned}
 &= 1 - 4 \sin A \sin B \cos C \\
 &= R.H.S.
 \end{aligned}$$

Q. If $A+B+C = \pi$, prove that

$$\begin{aligned}
 &\cos(4A) + \cos(4B) + \cos(4C) = \\
 &-1 + 4 \cos(2A) \cos(2B) \cos(2C)
 \end{aligned}$$

Ans: L.H.S.

$$= \cos(4A) + \cos(4B) + \cos(4C)$$

$$\begin{aligned}
 &= 2 \cos(2A+2B) \cos(2A-2B) + \cos(4C) \\
 &= 2 \cos(2\pi - 2C) \cos(2A-2B) + \\
 &\quad (2 \cos^2(2C) - 1)
 \end{aligned}$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ A+B = \pi - C \\ 2A+2B = 2\pi - 2C \end{array} \right.$$

$$\begin{aligned}
 &= 2 \cos(2C) \cos(2A-2B) + 2 \cos^2(2C) - 1 \\
 &= 2 \cos(2C) [\cos(2A-2B) + \cos(2C)] - 1
 \end{aligned}$$

$$= 2 \cdot \cos(2C) \cdot [\cos(2A-2B) + \cos(2\pi - (2A+2B))] - 1$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ 2A+2B+2C = 2\pi \\ 2C = 2\pi - (2A+2B) \end{array} \right\}$$

$$= 2 \cdot \cos(2C) \cdot [\cos(2A-2B) + \cos(2A+2B)] - 1$$

$$= 2 \cdot \cos(2C) \cdot [2 \cdot \cos(2A) \cdot \cos(2B)] - 1$$

$$= 4 \cdot \cos(2A) \cdot \cos(2B) \cdot \cos(2C) - 1$$

$$= R.H.S. \quad \text{proved} \Rightarrow$$

Q. Prove that :-

$$\text{If } A+B+C = \pi$$

$$\sin(A) + \sin(B) - \sin(C) = 4 \cdot \sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \cos\left(\frac{C}{2}\right)$$

Ans: L.H.S.

$$= (\sin(A) + \sin(B)) - \sin(C)$$

$$= 2 \cdot \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) - 2 \cdot \sin\left(\frac{C}{2}\right) \cdot \cos\left(\frac{C}{2}\right)$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ \frac{A+B+C}{2} = \frac{\pi}{2} \\ \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2} \end{array} \right\}$$

$$= 2 \cdot \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) - 2 \cdot \sin\left(\frac{C}{2}\right) \cdot \cos\left(\frac{C}{2}\right)$$

$$= 2 \cdot \cos\left(\frac{C}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) - 2 \cdot \sin\left(\frac{C}{2}\right) \cdot \cos\left(\frac{C}{2}\right)$$

$$= 2 \cdot \cos\left(\frac{C}{2}\right) \left[\cos\left(\frac{A-B}{2}\right) - \sin\left(\frac{C}{2}\right) \right]$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ \frac{A+B+C}{2} = \frac{\pi}{2} \\ \frac{C}{2} = \frac{\pi}{2} - \left(\frac{A+B}{2}\right) \end{array} \right.$$

$$= 2 \cdot \cos\left(\frac{C}{2}\right) \left[\cos\left(\frac{A-B}{2}\right) - \sin\left\{ \frac{\pi}{2} - \left(\frac{A+B}{2}\right) \right\} \right]$$

$$= 2 \cdot \cos\left(\frac{C}{2}\right) \cdot \left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right) \right]$$

$$= 2 \cdot \cos\left(\frac{C}{2}\right) \left[2 \cdot \sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \right]$$

$$= 4 \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{C}{2}\right) \cdot \sin\left(\frac{B}{2}\right)$$

$$= R \cdot H \cdot S$$

proved \rightarrow

Q. If $A+B+C = \pi$, prove that

$$\cos(A) + \cos(B) - \cos(C) = \left(4 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right)\right) - 1$$

Ans:
L.H.S.

$$= \cos(A) + \cos(B) - \cos(C)$$

$$= 2 \cdot \cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) - \left(1 - 2 \cdot \sin^2\left(\frac{C}{2}\right)\right)$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\ \Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2} \end{array} \right.$$

$$= 2 \cdot \cos\left(\frac{\pi}{2} - \frac{C}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) - \left(1 - 2 \cdot \sin^2\left(\frac{C}{2}\right)\right)$$

$$= 2 \cdot \sin\left(\frac{C}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) - 1 + 2 \sin^2\left(\frac{C}{2}\right)$$

$$= 2 \sin\left(\frac{C}{2}\right) \left[\cos\left(\frac{A-B}{2}\right) + \sin\left(\frac{C}{2}\right) \right] - 1$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\ \Rightarrow \frac{C}{2} = \frac{\pi}{2} - \left(\frac{A+B}{2}\right) \end{array} \right.$$

$$= 2 \cdot \sin\left(\frac{C}{2}\right) \left[\cos\left(\frac{A-B}{2}\right) + \sin\left\{\frac{\pi}{2} - \frac{(A+B)}{2}\right\} \right] - 1$$

$$= 2 \cdot \sin\left(\frac{C}{2}\right) \left[\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right) \right] - 1$$

$$= 2 \cdot \sin\left(\frac{C}{2}\right) \left\{ 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \right\} - 1$$

$$= \left\{ 4 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right) \right\} - 1$$

$$= R.H.S.$$

Q If $A+B+C = \pi$, prove that

$$\sin^2(A) + \sin^2(B) + \sin^2(C) = 2 \cdot (1 + \cos(A) \cdot \cos(B) \cdot \cos(C))$$

Ans:

L.H.S.

$$= \sin^2(A) + \sin^2(B) + \sin^2(C)$$

$$= \frac{1}{2} (1 - \cos(2A)) + \frac{1}{2} (1 - \cos(2B)) +$$

$$\frac{1}{2} (1 - \cos(2C))$$

$$= \frac{3}{2} - \frac{1}{2} (\cos(2A) + \cos(2B) + \cos(2C))$$

$$= \frac{3}{2} - \frac{1}{2} (-1 - 4 \cdot \cos A \cdot \cos B \cdot \cos C)$$

$$= 2 + 2 \cdot \cos A \cdot \cos B \cdot \cos C$$

$$= 2 (1 + \cos A \cdot \cos B \cdot \cos C)$$

$$= R.H.S.$$

Q. If $A+B+C = \pi$, prove that

$$\cos^2(A) + \cos^2(B) - \cos^2(C) = 1 - 2 \sin(A) \cdot \sin(B) \cdot \cos(C)$$

Ans:

L.H.S.

$$= \cos^2(A) + \cos^2(B) - \cos^2(C)$$

$$= \frac{1}{2} (1 + \cos(2A)) + \frac{1}{2} (1 + \cos(2B)) - \frac{1}{2} (1 + \cos(2C))$$

$$= \frac{1}{2} + \frac{1}{2} (\cos(2A) + \cos(2B) - \cos(2C))$$

$$= \frac{1}{2} + \frac{1}{2} (1 - 4 \sin(A) \cdot \sin(B) \cdot \cos(C))$$

$$= 1 - 2 \sin(A) \cdot \sin(B) \cdot \cos(C)$$

= R.H.S. proved

Q. In a $\triangle ABC$, prove that

$$\cos^2(A) + \cos^2\left(A + \frac{\pi}{3}\right) + \cos^2\left(A - \frac{\pi}{3}\right) = \frac{3}{2}$$

Ans:

L.H.S.

$$= \cos^2(A) + \cos^2\left(A + \frac{\pi}{3}\right) + \cos^2\left(A - \frac{\pi}{3}\right)$$

$$= \frac{1}{2} (1 + \cos(2A)) + \frac{1}{2} \left\{ 1 + \cos\left(2A + \frac{2\pi}{3}\right) \right\}$$

$$+ \frac{1}{2} \left\{ 1 + \cos\left(2A - \frac{2\pi}{3}\right) \right\}$$

$$= \frac{3}{2} + \frac{1}{2} \cdot \cos(2A) + \frac{1}{2} \left\{ \cos\left(2A + \frac{2\pi}{3}\right) + \cos\left(2A - \frac{2\pi}{3}\right) \right\}$$

$$= \frac{3}{2} + \frac{1}{2} \cdot \cos(2A) + \cos(2A) \cdot \cos\left(\frac{2\pi}{3}\right)$$

$$= \frac{3}{2} + \frac{1}{2} \cdot \cos(2A) + \cos(2A) \times -\frac{1}{2}$$

$$= \frac{3}{2} + \frac{1}{2} \cdot \cancel{\cos(2A)} - \frac{1}{2} \cdot \cancel{\cos(2A)}$$

$$= \frac{3}{2} = \text{R.H.S.} \quad \text{proved}$$

Q If $A+B+C = \pi$, prove that

$$\sin^2\left(\frac{A}{2}\right) + \sin^2\left(\frac{B}{2}\right) - \sin^2\left(\frac{C}{2}\right) = 1 - 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right)$$

Ans:

L.H.S.

$$= \sin^2\left(\frac{A}{2}\right) + \sin^2\left(\frac{B}{2}\right) - \sin^2\left(\frac{C}{2}\right)$$

$$= \frac{1}{2} (1 - \cos(A)) + \frac{1}{2} (1 - \cos(B)) - \frac{1}{2} (1 - \cos(C))$$

$$= \frac{1}{2} - \frac{1}{2} (\cos(A) + \cos(B) + \cos(C))$$

$$= \frac{1}{2} - \frac{1}{2} \left[\left(4 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right) \right) - 1 \right]$$

$$= 1 - 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right)$$

$= \text{R.H.S.} =$

Q. If $A+B+C=\pi$, prove that

$$\cos^2\left(\frac{A}{2}\right) + \cos^2\left(\frac{B}{2}\right) - \cos^2\left(\frac{C}{2}\right) = 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right)$$

Ans:

L.H.S.

$$= \cos^2\left(\frac{A}{2}\right) + \cos^2\left(\frac{B}{2}\right) - \cos^2\left(\frac{C}{2}\right)$$

$$= \frac{1}{2}(1+\cos(A)) + \frac{1}{2}(1+\cos(B)) - \frac{1}{2}(1+\cos(C))$$

$$= \frac{1}{2} + \frac{1}{2}(\cos(A) + \cos(B) - \cos(C))$$

$$= \frac{1}{2} + \frac{1}{2} \left[\left(4 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right) \right) - 1 \right]$$

$$= 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right)$$

= R.H.S.

Q. If $A+B+C=\pi$, prove that

$$\cos^2\left(\frac{A}{2}\right) + \cos^2\left(\frac{B}{2}\right) + \cos^2\left(\frac{C}{2}\right) = 2 \left(1 + \sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right) \right)$$

Ans:

L.H.S.

$$= \cos^2\left(\frac{A}{2}\right) + \cos^2\left(\frac{B}{2}\right) + \cos^2\left(\frac{C}{2}\right)$$

$$= \frac{1}{2}(1+\cos(A)) + \frac{1}{2}(1+\cos(B)) + \frac{1}{2}(1+\cos(C))$$

$$= \frac{3}{2} + \frac{1}{2}(\cos(A) + \cos(B) + \cos(C))$$

$$= \frac{3}{2} + \frac{1}{2} \left[2 \cdot \cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) + \cos C \right]$$

$$= \frac{3}{2} + \cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) + \frac{1}{2} \cos(C)$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\ \Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2} \end{array} \right.$$

$$= \frac{3}{2} + \cos\left(\frac{\pi}{2} - \frac{C}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) + \frac{1}{2} (1 - 2\sin^2\left(\frac{C}{2}\right))$$

$$= \frac{3}{2} + \sin\left(\frac{C}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right) + \frac{1}{2} - \sin^2\left(\frac{C}{2}\right)$$

$$= 2 + \sin\left(\frac{C}{2}\right) \left[\cos\left(\frac{A-B}{2}\right) - \sin\left(\frac{C}{2}\right) \right]$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\ \Rightarrow \frac{C}{2} = \frac{\pi}{2} - \left(\frac{A+B}{2}\right) \end{array} \right.$$

$$= 2 + \sin\left(\frac{C}{2}\right) \left[\cos\left(\frac{A-B}{2}\right) - \sin\left\{\frac{\pi}{2} - \left(\frac{A+B}{2}\right)\right\} \right]$$

$$= 2 + \sin\left(\frac{C}{2}\right) \left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right) \right]$$

$$= 2 + \sin\left(\frac{C}{2}\right) \left(2 \sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \right)$$

$$= 2 + 2 \cdot \sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right)$$

$$= 2 \left(1 + \sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right) \right)$$

$$= R.H.S \quad \text{proven}$$

Q. If $A+B+C = \pi$, prove that

$$\tan(A) + \tan(B) + \tan(C) = \tan(A) \cdot \tan(B) \cdot \tan(C)$$

Ans:

$$A+B+C = \pi$$

$$\Rightarrow A+B = (\pi - C)$$

$$\Rightarrow \tan(A+B) = \tan(\pi - C)$$

$$\Rightarrow \frac{\tan(A) + \tan(B)}{1 - \tan(A) \cdot \tan(B)} = -\tan(C)$$

$$\Rightarrow \tan(A) + \tan(B) = -\tan(C) + \tan(A) \cdot \tan(B) \cdot \tan(C)$$

$$\Rightarrow \tan(A) + \tan(B) + \tan(C) = \tan(A) \cdot \tan(B) \cdot \tan(C)$$

$$= R.H.S \quad \text{proven}$$

Q. If $A+B+C = \pi$, prove that

$$\tan\left(\frac{A}{2}\right) \cdot \tan\left(\frac{B}{2}\right) + \tan\left(\frac{B}{2}\right) \cdot \tan\left(\frac{C}{2}\right) + \tan\left(\frac{C}{2}\right) \cdot \tan\left(\frac{A}{2}\right) = 1.$$

Ans: $\left\{ \begin{array}{l} \text{here } A+B+C = \pi \\ \Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2} \end{array} \right\}$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) \\ = \cot\left(\frac{C}{2}\right)$$

$$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \cot\left(\frac{C}{2}\right)$$

$$\Rightarrow \frac{\tan\left(\frac{A}{2}\right) + \tan\left(\frac{B}{2}\right)}{1 - \tan\left(\frac{A}{2}\right) \cdot \tan\left(\frac{B}{2}\right)} = \frac{1}{\tan\left(\frac{C}{2}\right)}$$

$$\Rightarrow \tan\left(\frac{A}{2}\right) \cdot \tan\left(\frac{C}{2}\right) + \tan\left(\frac{B}{2}\right) \cdot \tan\left(\frac{C}{2}\right) = 1 - \tan\left(\frac{A}{2}\right) \cdot \tan\left(\frac{B}{2}\right)$$

$$\Rightarrow \tan\left(\frac{A}{2}\right) \cdot \tan\left(\frac{B}{2}\right) + \tan\left(\frac{B}{2}\right) \cdot \tan\left(\frac{C}{2}\right) + \tan\left(\frac{C}{2}\right) \cdot \tan\left(\frac{A}{2}\right) = 1.$$

proved

Q. If $A+B+C = \pi$, prove that

$$\cot(B) \cdot \cot(C) + \cot(C) \cdot \cot(A) + \cot(A) \cdot \cot(B) = 1$$

Ans: here,

$$A+B+C = \pi$$

$$\Rightarrow (A+B) = (\pi - C)$$

$$\Rightarrow \cot(A+B) = \cot(\pi - C)$$

$$\Rightarrow \cot(A+B) = -\cot(C)$$

$$\Rightarrow \frac{\cot(A) \cdot \cot(B) - 1}{\cot(A) + \cot(B)} = -\cot(C)$$

$$\Rightarrow \cot(A) \cdot \cot(B) - 1 = -\cot(A) \cdot \cot(C) - \cot(C) \cdot \cot(B)$$

$$\Rightarrow \cot(A) \cdot \cot(B) + \cot(B) \cdot \cot(C) + \cot(C) \cdot \cot(A) = 1$$

proved

Q. If $A+B+C = \pi$, prove that

$$\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right) = \cot\left(\frac{A}{2}\right) \cdot \cot\left(\frac{B}{2}\right) \cdot \cot\left(\frac{C}{2}\right)$$

Ans:

$$\text{here, } A+B+C = \pi$$

$$\Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2}$$

$$\Rightarrow \left(\frac{A}{2} + \frac{B}{2}\right) = \frac{\pi}{2} - \left(\frac{C}{2}\right)$$

$$\Rightarrow \cot\left(\frac{A}{2} + \frac{B}{2}\right) = \cot\left(\frac{\pi}{2} - \left(\frac{C}{2}\right)\right) = \tan\left(\frac{C}{2}\right)$$

$$\Rightarrow \frac{\cot\left(\frac{A}{2}\right) \cdot \cot\left(\frac{B}{2}\right) - 1}{\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right)} = \frac{1}{\cot\left(\frac{C}{2}\right)}$$

$$\Rightarrow \cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) = \cot\left(\frac{A}{2}\right) \cdot \cot\left(\frac{B}{2}\right) \cdot \cot\left(\frac{C}{2}\right) - \cot\left(\frac{C}{2}\right)$$

$$\Rightarrow \cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right) =$$

$$\cot\left(\frac{A}{2}\right) \cdot \cot\left(\frac{B}{2}\right) \cdot \cot\left(\frac{C}{2}\right)$$

provel

Q. If $A+B+C = \pi$, prove that

$$\sin(2A) + \sin(2B) - \sin(2C) = 4 \cos(A) \cos(B) \cos(C)$$

Ans:

L.H.S.

$$= \sin(2A) + \sin(2B) - \sin(2C)$$

$$= 2 \sin(A) \cos(A) + 2 \sin(B) \cos(B) - 2 \sin(C) \cos(C)$$

$$= 2 \sin(A) \cos(A) + 2 \sin(B) \cos(B) - 2 \sin(C) \cos(C)$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ B+C = \pi - A \\ \Rightarrow A = \pi - (B+C) \\ \Rightarrow B = \pi - (A+C) \\ \Rightarrow C = \pi - (A+B) \end{array} \right\}$$

$$= 2 \cdot \sin(\pi - (B+C)) \cdot \cos(A) +$$

$$2 \cdot \sin(\pi - (A+C)) \cdot \cos(C) -$$

$$2 \cdot \sin(\pi - (A+B)) \cdot \cos(A)$$

$$= 2 \cdot \sin(B+C) \cdot \cos(A) + 2 \cdot \sin(A+C) \cdot \cos(C) -$$

$$2 \cdot \sin(A+B) \cdot \cos(A)$$

$$= 2 (\sin(B) \cdot \cos(C) + \cos(B) \sin(C)) \cdot \cos(A) +$$

$$2 (\sin(A) \cdot \cos(C) + \cos(A) \sin(B)) \cdot \cos(C) -$$

$$2 (\sin(A) \cdot \cos(B) + \cos(A) \sin(B)) \cdot \cos(C)$$

$$= 2 \cdot \cos(A) \cdot \sin(B) \cdot \cos(C) + 2 \cos(A) \cdot \cos(B) \cdot \sin(C)$$

$$+ 2 \cdot \sin(A) \cdot \cos(C) \cdot \cos(C) + 2 \cdot \cos(A) \cdot \sin(C) \cdot \cos(B)$$

$$- 2 \cdot \sin(A) \cdot \cos(B) \cdot \cos(C) - 2 \cdot \cos(A) \cdot \sin(B) \cdot \cos(C)$$

$$= 2 \cdot \cos(A) \cdot \cos(B) \cdot \sin(C) +$$

$$2 \cdot \cos(A) \cdot \sin(B) \cdot \cos(B)$$

$$= 2 \cos(A) (\cos(B) \cdot \sin(C) + \sin(C) \cdot \cos(B))$$

$$= 2 \cos(A) \cdot (2 \cos(B) \cdot \sin(C))$$

$$= 4 \cdot \cos(A) \cdot \cos(B) \cdot \sin(C)$$

$= R.H.S.$ proved

Q. If $A+B+C = \pi$, prove that

$$\cos(2A) - \cos(2B) - \cos(2C) = -1 + 4 \cdot \cos(A) \cdot \sin(B) \cdot \sin(C)$$

Ans:

$$\because A+B+C=\pi$$

L.H.S.

$$= \cos(2A) - (\cos(2B) + \cos(2C))$$

$$= \cos(2A) - \left\{ 2 \cdot \cos\left(\frac{2B+2C}{2}\right) \cdot \cos\left(\frac{2B-2C}{2}\right) \right\}$$

$$= \cos(2A) - \left\{ 2 \cdot \cos(B+C) \cdot \cos(B-C) \right\}$$

$$\left\{ \begin{array}{l} \because A+B+C=\pi \\ \Rightarrow (B+C) = (\pi - A) \end{array} \right\}$$

$$= \cos(2A) - \left\{ 2 \cdot \cos(\pi - A) \cdot \cos(B-C) \right\}$$

$$= \cos(2A) - \left\{ -2 \cdot \cos(A) \cdot \cos(B-C) \right\}$$

$$= \cos(2A) + 2 \cdot \cos(A) \cdot \cos(B-C)$$

$$= 2 \cdot \cos^2(A) - 1 + 2 \cdot \cos(A) \cdot \cos(B-C)$$

$$= 2 \cdot \cos(A) \{ \cos(A) + \cos(B-C) \} - 1$$

$$= 2 \cdot \cos(A) \left\{ 2 \cdot \cos\left(\frac{A+B-C}{2}\right) \cdot \cos\left(\frac{A+C-B}{2}\right) \right\} - 1$$

$$= 2 \cdot \cos(A) \left\{ 2 \cdot \cos\left(\frac{\pi - C - C}{2}\right) \cdot \cos\left(\frac{\pi - B - B}{2}\right) \right\} - 1$$

$$= 2 \cdot \cos(A) \left\{ 2 \cdot \cos\left(\frac{\pi - 2C}{2}\right) \cdot \cos\left(\frac{\pi - 2B}{2}\right) \right\} - 1$$

$$= 2 \cdot \cos(A) \{ 2 \cdot \sin(C) \cdot \sin(B) \} - 1$$

$$= 4 \cdot \cos(A) \cdot \sin(B) \cdot \sin(C) - 1$$

$$= R.H.S.$$

Q If $A+B+C = \pi$, prove that :-

$$\cos(2A) - \cos(2B) + \cos(2C) = 1 - 4 \cdot \sin(A) \cdot \cos(B) \cdot \sin(C)$$

Ans: L.H.S.

$$= \cos(2A) - \cos(2B) + \cos(2C)$$

$$= \cos(2A) - \left\{ 2 \cdot \sin\left(\frac{2B+2C}{2}\right) \cdot \sin\left(\frac{2B-2C}{2}\right) \right\}$$

$$= \cos(2A) - \left\{ 2 \cdot \sin(B+C) \cdot \sin(B-C) \right\}$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ B+C = (180^\circ - A) \end{array} \right\}$$

$$= \cos(2A) - \left\{ 2 \cdot \sin(\pi - A) \cdot \sin(B-C) \right\}$$

$$= \cos(2A) - \left\{ 2 \cdot \sin(A) \cdot \sin(B-C) \right\}$$

$$= 1 - 2 \cdot \sin^2 A - 2 \sin(A) \cdot \sin(B-C)$$

$$= -2 \cdot \sin(A) \{ \sin A + \sin(B-C) \} + 1$$

$$= -2 \cdot \sin(A) \left\{ 2 \cdot \sin\left(\frac{A+B-C}{2}\right) \cdot \cos\left(\frac{A+C-B}{2}\right) \right\} + 1$$

$$\left. \begin{aligned} &= -2 \cdot \sin(A) \left\{ \begin{aligned} &\because A+B+C = \pi \\ &\Rightarrow A+B = (\pi - C) \\ &\Rightarrow A+C = (\pi - B) \end{aligned} \right. \end{aligned} \right\}$$

$$= -2 \cdot \sin(A) \left\{ 2 \cdot \sin\left(\frac{\pi - C - C}{2}\right) \cdot \cos\left(\frac{\pi - B - B}{2}\right) \right\} + 1$$

$$= -2 \cdot \sin(A) \left\{ 2 \cdot \sin\left(\frac{\pi}{2} - \frac{2C}{2}\right) \cdot \cos\left(\frac{\pi}{2} - \frac{2B}{2}\right) \right\} + 1$$

$$= -2 \cdot \sin(A) \left\{ 2 \cdot \sin\left(\frac{\pi}{2} - C\right) \cdot \cos\left(\frac{\pi}{2} - B\right) \right\} + 1$$

$$= -2 \cdot \sin(A) \left\{ 2 \cdot \cos(C) \cdot \sin(B) \right\} + 1$$

$$= -4 \cdot \sin(A) \cdot \sin(B) \cdot \cos(C) + 1$$

$$= R.H.S.$$

proven

Q. If $A+B+C=\pi$, prove that

$$\sin(A) + \sin(B) + \sin(C) = 4 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \cos\left(\frac{C}{2}\right)$$

Ans. L.H.S.

$$= \sin(A) + \sin(B) + \sin(C)$$

$$= \sin(A) + \left\{ 2 \cdot \sin\left(\frac{B+C}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$\# \left\{ \begin{array}{l} \because A+B+C=\pi \\ \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \end{array} \right.$$

$$\Rightarrow \frac{B+C}{2} = \left(\frac{\pi}{2} - \frac{A}{2} \right)$$

$$= \sin(A) + \left\{ 2 \cdot \sin\left(\frac{\pi}{2} - \frac{A}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 2 \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{A}{2}\right) + \left\{ 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 2 \cdot \cos\left(\frac{A}{2}\right) \left\{ \sin\left(\frac{A}{2}\right) + \cos\left(\frac{B-C}{2}\right) \right\}$$

$$\left\{ \begin{array}{l} \because A+B+C=\pi \\ \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\ \Rightarrow \frac{A}{2} = \left\{ \frac{\pi}{2} - \left(\frac{B+C}{2}\right) \right\} \end{array} \right.$$

$$\begin{aligned}
 &= 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \left\{ \sin\left\{\frac{\pi}{2} - \left(\frac{B+C}{2}\right)\right\} + \cos\left(\frac{B-C}{2}\right) \right\} \\
 &= 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \left\{ \cos\left(\frac{B+C}{2}\right) + \cos\left(\frac{B-C}{2}\right) \right\} \\
 &= 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \left\{ 2 \cdot \cos\left(\frac{B}{2}\right) \cdot \cos\left(\frac{C}{2}\right) \right\} \\
 &= 4 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \cos\left(\frac{C}{2}\right) \\
 &= R \cdot H \cdot S \quad \text{proved}
 \end{aligned}$$

Q. If $A+B+C = \pi$, prove that

$$\cos(A) + \cos(B) + \cos(C) =$$

$$1 + 4 \cdot \sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right)$$

Ans: L.H.S.

$$= \cos(A) + \cos(B) + \cos(C)$$

$$= \cos(A) + \left\{ 2 \cdot \cos\left(\frac{B+C}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$\left\{ \begin{aligned}
 &\because A+B+C = \pi \\
 &\Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\
 &\Rightarrow \frac{B+C}{2} = \left(\frac{\pi}{2} - \frac{A}{2}\right)
 \end{aligned} \right.$$

$$= \cos(A) + \left\{ 2 \cdot \cos\left(\frac{\pi-A}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= \cos(A) + \left\{ 2 \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 1 - 2 \cdot \sin^2\left(\frac{A}{2}\right) + \left\{ 2 \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 2 \cdot \sin\left(\frac{A}{2}\right) \left\{ -\sin\frac{A}{2} + \cos\left(\frac{B-C}{2}\right) \right\} + 1$$

$$= 2 \cdot \sin\left(\frac{A}{2}\right) \cdot \left\{ \cos\left(-\frac{B-C}{2}\right) + \cos\left(\frac{B-C}{2}\right) \right\} + 1$$

$$= 2 \cdot \sin\left(\frac{A}{2}\right) \cdot \left\{ 2 \cdot \cos\left(-\frac{C}{2}\right) \cdot \cos\left(-\frac{B}{2}\right) \right\} + 1$$

$$= 4 \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \cos\left(\frac{C}{2}\right) + 1$$

∴ R.H.S. = L.H.S. proved

Q. If $A+B+C = \pi$, prove that :-

$$\frac{\sin(2A) + \sin(2B) + \sin(2C)}{\sin(A) + \sin(B) + \sin(C)} = \frac{8 \cdot \sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right)}{\sin\left(\frac{C}{2}\right)}$$

Ans: L.H.S.

1st part

$$\sin(2A) + \sin(2B) + \sin(2C)$$

$$= 2 \sin A \cos A + 2 \sin(B+C) \cos(B-C)$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ \Rightarrow (B+C) = \pi - A \end{array} \right\}$$

$$= 2 \sin A \cos A + 2 \sin(\pi - A) \cos(B-C)$$

$$= 2 \sin A \cos A + 2 \sin A \cos(B-C)$$

$$= 2 \sin A \{ \cos A + \cos(B-C) \}$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ A = \pi - (B+C) \end{array} \right\}$$

$$= 2 \sin A \{ \cos(\pi - (B+C)) + \cos(B-C) \}$$

$$= 2 \sin A \{ \cos(B+C) + \cos(B-C) \}$$

$$= 2 \sin A \times \{ 2 \cos(B+C) - \cos(C-B) \}$$

$$= 2 \sin A \{ 2 \sin B \sin C \}$$

$$= 4 \sin A \sin B \sin C$$

$$= 4 \times \left\{ 2 \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{A}{2}\right) \right\} \times \left\{ 2 \sin\left(\frac{B}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \right\} \times \left\{ 2 \sin\left(\frac{C}{2}\right) \cdot \cos\left(\frac{C}{2}\right) \right\}$$

$$= 32 \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right) \cdot \cos\left(\frac{C}{2}\right)$$

2nd part

$$= \sin(A) + \sin(B) + \sin(C)$$

$$= \sin(A) + \left\{ 2 \cdot \sin\left(\frac{B+C}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= \sin(A) + \left\{ 2 \cdot \sin\left(\frac{\pi-A}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$\left\{ \begin{array}{l} \because A+B+C = \pi \\ \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\ \Rightarrow \frac{B+C}{2} = \frac{\pi}{2} - \frac{A}{2} \end{array} \right\}$$

$$= \sin(A) + \left\{ 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 2 \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{A}{2}\right) + \left\{ 2 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 2 \cdot \cos\left(\frac{A}{2}\right) \left\{ \sin\left(\frac{A}{2}\right) + \cos\left(\frac{B-C}{2}\right) \right\}$$

~~2~~ $\left\{ \begin{array}{l} \because A+B+C = \pi \\ \Rightarrow \frac{A+B+C}{2} = \frac{\pi}{2} \\ \Rightarrow \frac{A}{2} = \frac{\pi}{2} - \left(\frac{B+C}{2}\right) \end{array} \right\}$

$$= 2 \cdot \cos\left(\frac{A}{2}\right) \left\{ \sin\left(\frac{\pi}{2} - \left(\frac{B+C}{2}\right)\right) + \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 2 \cdot \cos\left(\frac{A}{2}\right) \left\{ \cos\left(\frac{B+C}{2}\right) + \cos\left(\frac{B-C}{2}\right) \right\}$$

$$= 2 \cdot \cos\left(\frac{A}{2}\right) \left\{ 2 \cdot \cos\left(\frac{B}{2}\right) \cdot \cos\left(\frac{C}{2}\right) \right\}$$

$$= 4 \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \cos\left(\frac{C}{2}\right)$$

therefore,

L.H.S.

$$= \frac{\text{Ist part}}{\text{IInd part}}$$

$$\begin{aligned}
 & \frac{8}{32} \cdot \sin\left(\frac{A}{2}\right) \cdot \cos\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right) \cdot \cos\left(\frac{C}{2}\right) \\
 &= \frac{1}{4} \cdot \cos\left(\frac{A}{2}\right) \cdot \cos\left(\frac{B}{2}\right) \cdot \cos\left(\frac{C}{2}\right) \\
 &= 8 \cdot \sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right) \\
 &= R \cdot H \cdot S \quad \text{proved.}
 \end{aligned}$$

Q. If $A+B+C = \pi$, prove that

$$\begin{aligned}
 & \sin(B+C-A) + \sin(C+A-B) - \sin(A+B-C) \\
 &= 4 \cdot \cos(A) \cdot \cos(B) \cdot \cos(C)
 \end{aligned}$$

Ans: L.H.S.

$$\begin{aligned}
 &= \sin(B+C-A) + \sin(C+A-B) - \sin(A+B-C) \\
 &= 2 \cdot \sin(C) \cdot \cos(B-A) - \sin(A+B-C) \\
 &\quad \left\{ \begin{array}{l} \because A+B+C = \pi \\ \Rightarrow B+A = \pi - C \end{array} \right\} \\
 &= 2 \cdot \sin C \cdot \cos(B-A) - \sin(\pi - C - C) \\
 &= 2 \cdot \sin C \cdot \cos(B-A) - \sin 2C \\
 &= 2 \cdot \sin C \cdot \cos(B-A) - 2 \cdot \sin C \cdot \cos C \\
 &= 2 \cdot \sin C \{ \cos(B-A) - \cos C \} \\
 &= 2 \cdot \sin C \left\{ 2 \sin\left(\frac{B-A+C}{2}\right) \cdot \sin\left(\frac{C-B+A}{2}\right) \right\}
 \end{aligned}$$

$$= 2 \cdot \sin C \cdot \left\{ 2 \sin \left(\frac{\pi - A - A}{2} \right) \cdot \sin \left(\frac{\pi - B - B}{2} \right) \right\}$$

$$= 4 \cdot \cos A \cdot \cos B \cdot \sin C$$

$$= R \cdot H \cdot S$$

Proved

total word, $\pi = A + B + C$ (1)

$$- (A - B + C) \sin \frac{A}{2} + (A - C + B) \sin \frac{B}{2} - (B - C + A) \sin \frac{C}{2}$$

$$(1) (2) \cdot (3) (4) \cdot (A) (B) \cdot A =$$

$$2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$(1) (2) \sin \frac{A}{2} - (A - B + C) \sin \frac{A}{2} + (A - C + B) \sin \frac{B}{2} =$$

$$(1 - B + A) \sin \frac{A}{2} - (A - B) \sin \frac{B}{2} + (A - C) \sin \frac{C}{2} =$$

$$\left\{ \begin{aligned} &1 - B + A \\ &A - B \\ &A - C \end{aligned} \right\}$$

$$(1 - B + A) \sin \frac{A}{2} - (A - B) \sin \frac{B}{2} + (A - C) \sin \frac{C}{2} =$$

$$(1 - B + A) \sin \frac{A}{2} - (A - B) \sin \frac{B}{2} + (A - C) \sin \frac{C}{2} =$$

$$(1 - B + A) \sin \frac{A}{2} - (A - B) \sin \frac{B}{2} + (A - C) \sin \frac{C}{2} =$$

$$(1 - B + A) \sin \frac{A}{2} - (A - B) \sin \frac{B}{2} + (A - C) \sin \frac{C}{2} =$$